Learning to teach and teaching to learn

Mike Ollerton and Kirsty Cooper describe their underpinning principles for writing a scheme of work and share some mathematical activities.

Over the past eighteen months, I have had the pleasure of working with a Y3 to Y6 class on a voluntary basis, once or twice a week. The school is Grayrigg, my local primary in South Cumbria. Working with children of this wide age range has taught me a lot, particularly about the importance of expectations, the value of surprises and of being surprised.

Initially, the headteacher, Kirsty, gave me carte-blanche to try out different ideas with her class and, being aware of the wide age range, I sought to offer tasks with starting points that were accessible to all the children.

Here are some of the tasks I used:

Triangles on a 9-pin geoboard

Quite simply, I took enough 9-pin geoboards for one between pairs of children and some grid paper for recording and asked them to make different triangles. This is a task I have used many times in the past. Pedagogically it is accessible to all learners. It also enabled interesting discussion about what the word 'different' meant, which led to introducing the concepts of congruence and similarity.

Rectangles, dimensions, areas and perimeter

I gave everyone a small piece of 1cm squared grid paper and asked them to draw a rectangle on it. Then I asked them to record the length and the height, find its area and its perimeter. Everyone could find the area of their rectangle, although I noticed some had put a dot in each square. This led to a discussion about how they could calculate the area of their rectangle without counting each square individually.

Working out the perimeter of their rectangle, however, proved more complicated. Some had correctly worked out their perimeter. Others counted the number of internal squares inside each edge. For example, a 6 by 3 rectangle produced an incorrect perimeter answer of 14:

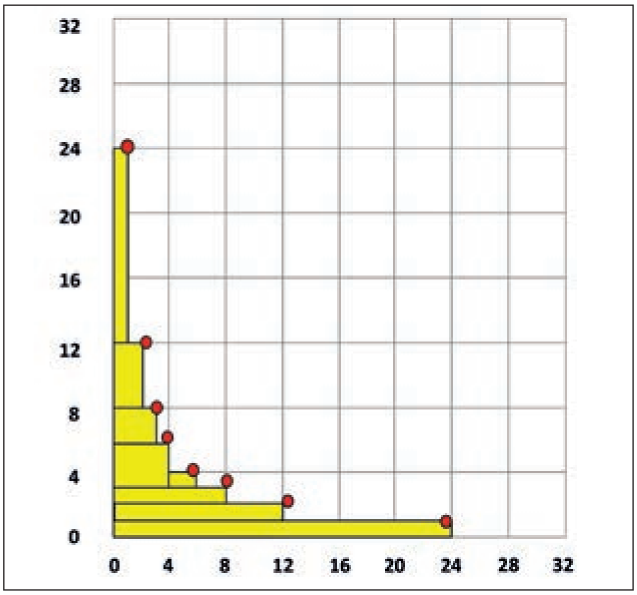
1	2	3	4	5	6
14					7
13	12	11	10	9	8

Others meanwhile had counted the surrounding square, arriving at a different incorrect answer.

1	2	3	4	5	6	7	8
22							9
21							10
20							11
19	18	17	16	15	14	13	12

Both methods led to useful discussions about what a perimeter is and how it can be calculated. The next part of the lesson was to ask everyone to make a sketch of their rectangle, without drawing the actual squares, on a piece of sugar paper and to add two pieces of information taken from length, width, area and perimeter. I also asked them to write their name on the sugar paper. The idea was for them to stick their completed problem on a wall to then go around the room working out the missing information from each others' pieces of work.

A development involved drawing some rectangles with a fixed area of 24cm^2 on 1cm square grid paper. As they drew their rectangles, I asked them to record what the different dimensions were, for example, 2 by 12, 6 by 4. Some children cut out their rectangles and stuck them on a pair of axes (see below).



Others turned their dimensions into co-ordinate pairs and drew these on graph paper. Some children were challenged to think about dimensions which were not integer values such as $1\frac{1}{2}$ and 16, and plotting the point ($1\frac{1}{2}$, 16). Telling them the name of the resulting curve was a hyperbola was a purposeful decision. Later, when I asked them to write about and/or draw pictures, or make a presentation of what they had been doing, I noticed one Y4 girl had looked for a picture of a hyperbola on the internet to copy and paste it into her presentation. This proved to be the beginning of thinking about getting children to write journals, for them to explain in greater depth what all this 'doing' of mathematics had been about as far as they were concerned. What had they understood? What was the 'best' problem they had solved?

In another lesson we drew rectangles with a fixed perimeter of 20cm, and this led to them drawing two graphs, one of which was a straight line graph with co-ordinates (1,9), (2,8), (3,7) and so on and another a negative quadratic, when one of the dimensions was matched with area, for example, 9, 16, 21, 24, 25, 24 emerging from (1, 9), (2, 16), (3, 21) and so on.

We played the game of 4-in-a line using pegs and pegboards; recording the winning lines as four pairs of co-ordinates, looking for the number patterns, predicting more points at either end of their winning lines and having conversations with some of the children about the names of some of the lines.

We did lots of work on "guzintas" (you know, what goes into 20?) which of course I later named as divisors, we did some dancing: we became a clock face. All the time I was getting to know the children, noticing how they worked so easily in pairs or small groups and gaining a sense of them enjoying mathematics.

Given the children's okay-ness with drawing axes and plotting co-ordinates, my mind turned to a more substantial task Kirsty and I wanted to work on in preparation for September 2016; this was to rework the KS2 scheme of work. We began by discussing a set of principles upon which we wanted the new SoW to be based. These were/are:

1. Units of work should last at least three weeks to enable children to focus on both developing skills and understanding mathematical structures. The skills should include those of reasoning, conjecturing and developing generality.
2. Each unit should provide a series of tasks, progressive both in terms of curriculum demands and children's cognition.
3. Each unit should develop a range of ways of working to include the use of manipulatives, IT, calculators, mental imagery, discussion, collaboration and independent work.
4. Within each unit, children should be encouraged to write journals, illustrated with pictures/photos and to use electronic media to explain aspects of their learning that they are confident about and are proud of to share with others.
5. Looking to see what different types of representations we can encourage children to consider for any mathematics they work on.
6. Encouraging children to look for connections between different units so they come to view mathematics as an interconnected whole.
7. To actively slow-down in order to deepen children's cognition; certainly not seeking to accelerate cognition. Learning mathematics must not be seen as a 'race'.
8. We cannot do the learning for the children.

Since the end of the 2015-16 academic year, we have been planning to be more systematic in developing a KS2 scheme of work based upon these principles. This resulted in us deciding upon key ideas/units of work which are:

a) place value

b) addition & subtraction

c) multiplication & division

d) fractions

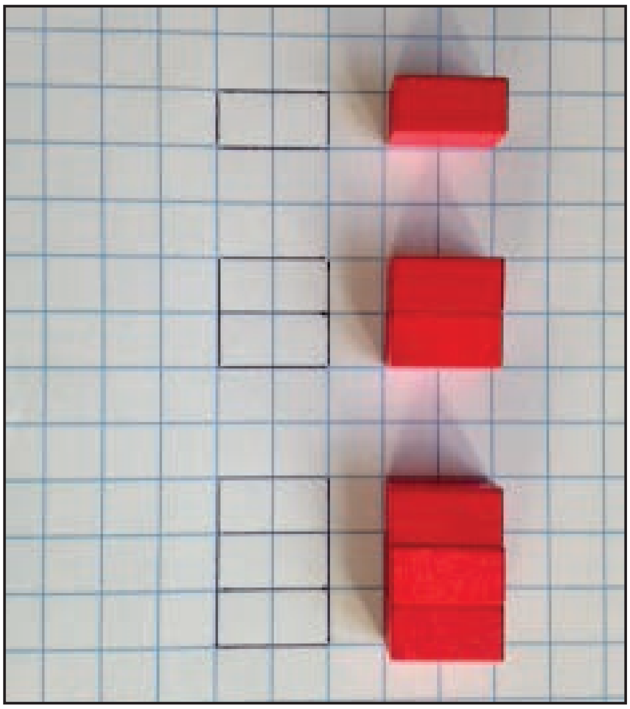
e) geometry.

We decided that the National Curriculum units pertaining to measures, algebra and statistics would be integrated within a), b), c), d) and e) above. Work on sequences, co-ordinates and graphs would be an important aspect of multiple representations (see principle 5 above). The idea of multiple representations is nothing new. More recently though they have, rightly in my view, been given a lot of attention as a way of helping learners make sense of mathematical structure.

Below are some examples of tasks and the different types of representations we can ask children to work on. They are both sets of tasks that could easily fit into a unit of work on multiplication and division and are based upon an accessible starting point of the multiples of two. The starting point is to explore different ways the sequence 2, 4, 6, 8, … might be represented. So far we have the following four types of representations:

i) As a sequence of multiplication facts, i.e., 1 x 2 = 2, 2 x 2 = 4, 3 x 2 = 6 and so on.

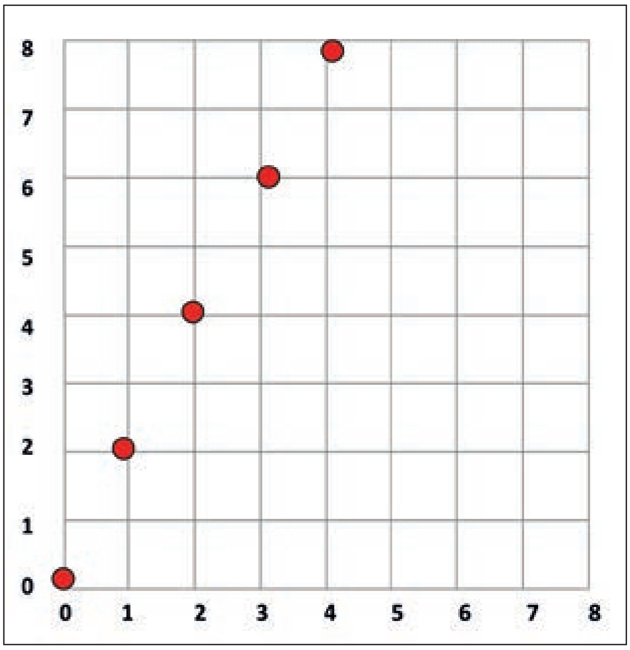
ii) With red Cuisenaire rods, all of which are two high.



iii) Drawing rectangles on square grid paper and calculating the area of each one.

iv) As a set of co-ordinates on a grid. To do this we can plot the points (1, 2), (2, 4), (3, 6) and so on.

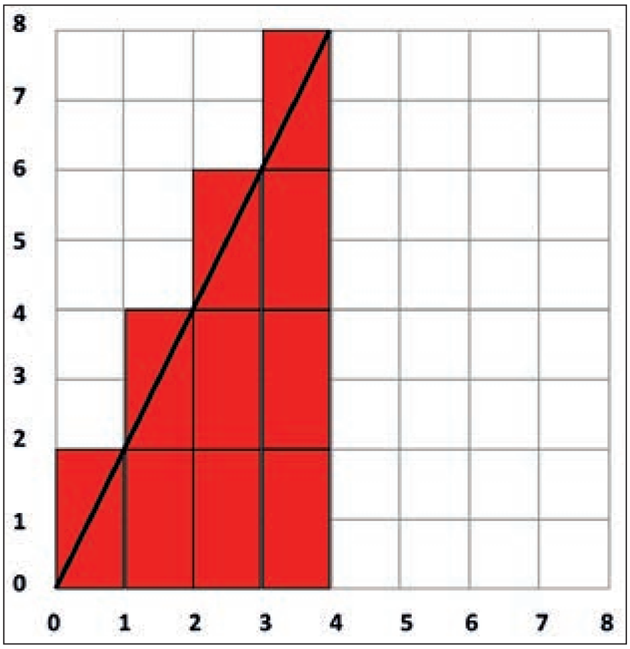
For sake of completeness, I will add the point (0, 0) and this will become a point for discussion.



Further questions could be:

- What does the graph of the multiples of 3 look like?
- What does the graph of the multiples of 1 look like?

The multiples of 2 graph can also be created using red Cuisenaire rods as follows:



Returning to linear sequences as a development of the multiples of 2 we can look at the sequence of odd numbers; 1, 3, 5, 7…

As above, the same structures and their representations can be applied:

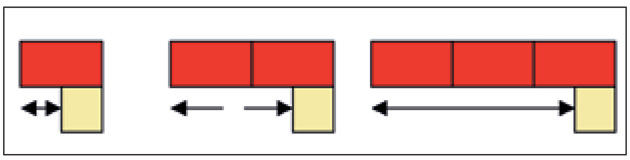
i) As a sequence of multiplication facts, i.e., 1 x 2 – 1 = 1,

2 x 2 – 1 = 3

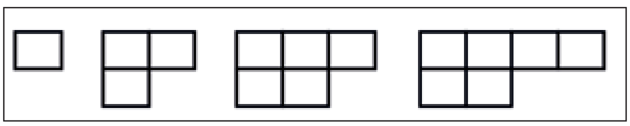
3 x 2 – 1 = 5 and so on.

ii) With Cuisenaire rods:

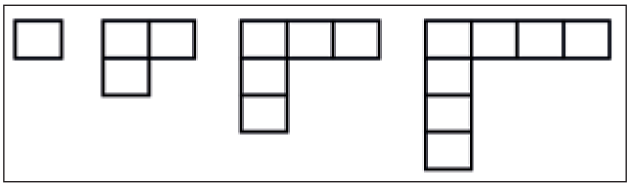
involving looking at the difference shown by the gaps:



iii) Making pictures:

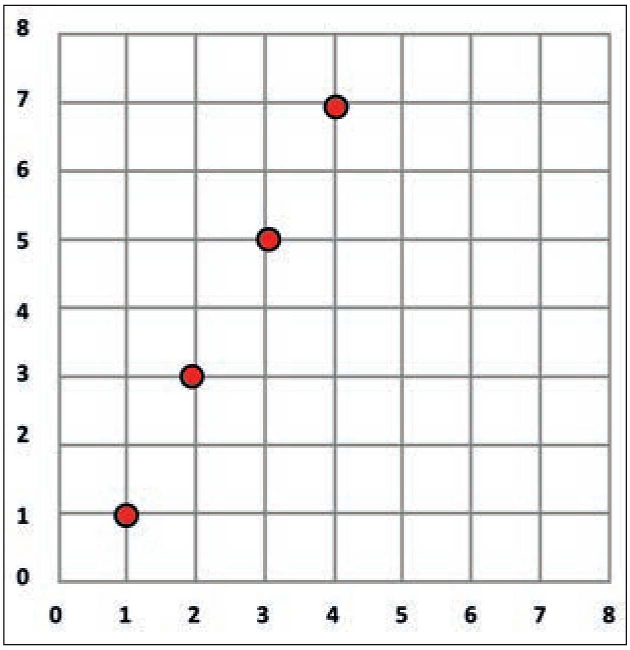


or



iv) As a set of co-ordinates on a grid:

(1, 1), (2, 3), (3, 5), (4, 7) and so on.

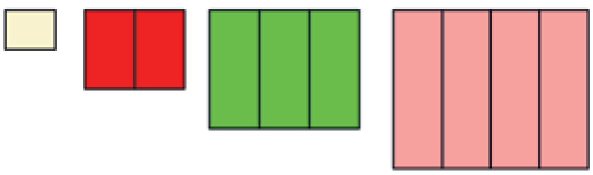


Of course we do not need to stop at linear sequences.

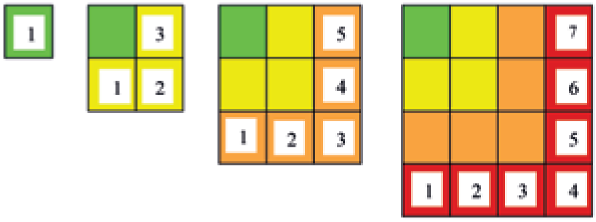
What does the square number sequence: 1, 4, 9, 16, look like in different representations?

i) As a set of multiplication facts: 1 x 1, 2 x 2, 3 x 3, 4 x 4, and so on.

ii) With Cuisenaire rods:



iii) As squares, which when built up as shown below illustrates one of the main properties of the square number sequence, 1, 1 + 3, 1 + 3 + 5, 1 + 3 + 5 + 7… the sum of consecutive odd numbers starting with 1.

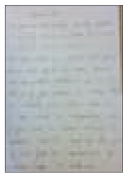


iv) As a set of co-ordinate pairs on a grid: (0, 0), (1, 1), (2, 4), (3, 9)…

With last year's KS2 group this set of numbers had previously emerged from some work about the leading diagonal in a multiplication grid. As such, square numbers were not new to the class. What was new, or in its infancy, was the idea of journal writing, principle 4, which will be embedded in each unit of work.

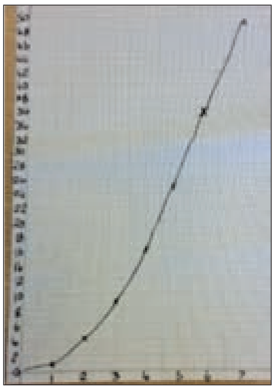
Below are two pages of Samara's (age 11) written work. Here she explains the work she had done on square numbers leading to square roots, or 'unsquaring' numbers, which formed part of a further challenge task.

Square Roots



First you list times numbers starting from 1 (1x1 = 2, 2x2 = 4, 3x3 = 9, 4x4 = 16, 5x5 = 25). Then you make a graph that goes up in twos to 50 and space out the bottom numbers up to 7. Then if you choose a number that has already got a dot or a star on it (on the y-axis) that would be unsquaring and if you pick a number between 2 numbers would be squaring this you try to find the square root of whatever number is between.

Although Samara's text does not make complete sense, when she explained the process of creating the graph then using it to 'unsquare' numbers to me, it was clear she understood the key ideas of squaring and square-rooting.



As it happened, this was only the second piece of mathematical journal writing the class had done. We encouraged the Y6 pupils to think of their audience as their, soon to be, new Y7 teacher.

We are convinced of the value of journal writing, of expecting children to be explicit about what they have been doing, and what they have understood as an outcome of all this ‘doing’, to describe parts of the (mathematical) journeys they have undertaken. Currently (September 2016), we have just begun the third week of work on place value. We have planned a series of challenge activities. None of the children are anywhere near completing all these challenges yet. This is okay, because there will be at least one further place value unit in the next two terms. Within these units will be deeper opportunities for pattern-spotting, conjecturing and generalising.

Gaining access to lots of ideas which we can weave into our teaching is a paramount consideration. Teachers new to the profession or those who have been either inspired or drained by national strategies or other national initiatives may benefit more by purchasing ATM publications where we will find all kinds of accessible, problem-based, ideas to take with us into classrooms; ideas which we can take ownership of and be proud to share with colleagues.

Next week we will begin a three-week geometry based unit of work. What joy.

Mike Ollerton is passionate about mathematics teaching, problem solving, mixed-ability, fell walking, cycling, Liverpool football club and dearly-beloved. Not necessarily in that order.

Kirsty Cooper is the head teacher at Grayrigg Primary School.