

The Shanghai exchange: Reflections from a UK perspective

Sue Pope and Mike Ollerton reflect on the implications for teaching and learning of their experiences when spending a day with two teachers from Shanghai.

There has been a teacher exchange between England and Shanghai. In 2014-15 this involved primary schools and in 2015-16 secondary schools. We had the opportunity to spend a day with two teachers from Shanghai in a south Manchester 11-18 mixed comprehensive school.

Prior to the visit, we were asked to read *Teaching with procedural variation: A Chinese way of promoting deep understanding of mathematics* by Lai and Murray (2012). The abstract includes:

Western educators often emphasise the need for students to construct a conceptual understanding of mathematical symbols and rules before they practise the rules (Li, 2006). On the other hand, Chinese learners tend to be oriented towards rote learning and memorisation (Marton, Watkins & Tang, 1997). One aspect of the criticism is that rote learning is known to lead to poor learning outcomes (Watkins & Biggs, 2001). However, Chinese students consistently outperform their Western counterparts in many international comparative studies on mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1997; Mullis, Martin, & Foy, 2008) and PISA (OECD, 2004; OECD, 2010).

They describe China's success in international comparisons as the 'Chinese Paradox' and make extensive links between procedural variation with the work on task design by Watson and Mason (2005, 2006).

During their two-week visit the teachers had taught a sequence of lessons each, one on fractions with Y8 and the other on decimal division and the mean with Y7. We saw one of each lesson with pre- and post-lesson discussion. The class teacher was with the classes and assisted with classroom management.

The teachers had a clear rationale for their sequence of lessons with a detailed breakdown of what happens when. The teaching of fractions was based on Chapter 2 of the Shanghai text book for grade 6 (equivalent to UK Year 7) which systematically introduces:

- fractions related to division. Understanding that when a length of 4m is divided into five equal pieces, each piece is $\frac{1}{5}$ of the original with a

length of $\frac{4}{5}$ m;

- equivalent fractions;
- ordering fractions;
- simplifying fractions and the simplest form;
- addition including mixed numbers and subtraction.

Six to eight weeks are spent on this work that is preceded by a chapter on lowest common multiple and highest common factor, essential skills for comparing, ordering, adding and subtracting fractions. Whilst children meet fractions in primary school they will not have done any of this work before. In Shanghai, they will not repeat this formal work again. It becomes 'assumed knowledge'. This is because of the belief that once taught 'properly' a topic does not need to be taught again, as students are expected to take responsibility for maintaining familiarity with what has been taught.

In the lesson we observed, mixed numbers were being subtracted where the fractional part of the number being subtracted was greater than the fractional part of the other number, for example $7\frac{1}{9} - 3\frac{8}{9}$ with a development to using fractions with different denominators: $7\frac{1}{9} - 3\frac{8}{11}$.

The fractions lesson began with a review of homework in which the formal presentation of work and the correction of errors were emphasised. Students were invited to work on $7\frac{1}{9} - 3\frac{8}{9}$, after several minutes the teacher went through three methods, drawing on student feedback:

$$M_1: (7 - 3) + \left(\frac{1}{9} - \frac{8}{9}\right) = 4 - \frac{7}{9}$$

$$M_2: \frac{64}{9} - \frac{35}{9} = \frac{29}{9}$$

$$M_3: \left(6 + \frac{9}{9} + \frac{2}{9}\right) - \frac{38}{9} = (6 - 3) + \frac{11}{9} - \frac{8}{9}$$

The class was divided into four mixed groups and a comparable question was set for each group. The groups select a different representative each time to write up the solution on the board. This approach to working had been established with the class and was completed fairly quickly. The teacher then reviewed the four responses with a red pen asking which method had been used and clarifying where formal recording was inaccurate or the equals sign had been used incorrectly. The vast majority of students paid careful attention to

this review. At the end of each response a mark out of 1 was awarded for both correct arithmetic and formal recording. A correct answer scored 1 with marks lost for incorrect use of mathematical formalism (they get a decimal answer) this was negotiated with the class.

The class was then asked to work on the planned development: $7\frac{1}{9} - 3\frac{8}{11}$. Students were asked to articulate in what way this problem was different and then left to decide how to work on the problem. The choice of 11 and the need to use 99ths, which several of the students noticed, made method M_2 impractical. The answer was then worked through with the class before another exercise was set for the groups. This time two of the questions did not have co-prime denominators (the denominators had a common factor) and one had both the whole number part and fractional part less than in the greater number for example $6\frac{3}{4} - 2\frac{2}{3}$.

We discussed the choice of examples after the lesson. The teacher was clear that this item was included so students knew that subtraction might involve the fractional part less than in the number being subtracted from. Again, a student from each group wrote up the answer and the teacher reviewed the answers using a red pen and scored them with class involvement. The scores from the exercises were added to scores achieved in previous lessons and the students were asked to give the total scores, thus mental addition of decimals was also incorporated into the lesson. At the end of the lesson the board contained a powerful summary of what had been done.

Whilst many in the class were engaged with the lesson, there were some who completed work quickly and spent a lot of time unoccupied and there were some who did not appear to have much of a clue about what was going on. We discussed this after the lesson. The teacher explained that in Shanghai lessons are pitched 'at the middle'. There is a mathematics competition book, which is used to provide challenge to more confident

students and there is always a homework exercise, which students can get on with if there is time available in lessons. All students know to record everything recorded on the board during the lesson. Less confident students receive individual support from their teachers at the end of the school day (07:30 to 16:30) in mathematics, English and Chinese. Teachers may not finish giving additional support until 19:00. Teachers are responsible for teaching six 40-minute lessons each week to two groups of approximately 40 students, so they teach two or three lessons a day. Some teachers supervise the last lesson of the day where students work on exercises related to the day's lessons. In addition to the after-school support during the week, most students attend private classes at the weekends.

The second lesson we observed was on the mean. The lesson started with a review of homework and a discussion of different approaches to finding the mean:

- Method 1: adding all and dividing by the number of items.
- Method 2: making all the data values equal by 'moving from the greater to the lesser'.
- Method 3: taking the lowest number as a baseline ('general level') and finding the average of the additional amounts.
- Method 4: taking the most common number as a baseline and finding the average of the differences.

The theme of the lesson was method 4. Throughout the sequence of lessons there had been an emphasis on understanding the mean must lie between the minimum and maximum values. A set of values ranging between 196 and 203 with several repeats of 200 was given to the students. One student computed the mean using method 3 quickly, a few others attempted to use method 4. The teacher gave a quick demonstration of method 4 and said 'finish this method', then promptly cleaned

$7\frac{1}{9} - 3\frac{8}{9}$ with working M_1, M_2 and M_3	$7\frac{1}{9} - 3\frac{8}{11}$ with working		Group scores
Exercise 1			
Question with student working and teacher review.	Question with student working and teacher review.	Question with student working and teacher review.	Question with student working and teacher review.
Exercise 2			
Question with student working and teacher review.	Question with student working and teacher review.	Question with student working and teacher review.	Question with student working and teacher review.

the board. Approximately ten minutes later, when many students still had not managed to compute the mean, the teacher went through method 4 again, this time involving the students at a much slower pace.

A further set of data was given: 38, 0, 52, 21, 0, 24 and students were asked which method to use. Students said method 1 and were able to complete it for this small set of data.

The students had generated their own data in a practical lesson on the previous day and the teacher said they must work on that data for homework. This caused some consternation as they had worked in groups and only one person had the data. The class teacher intervened and told them to take photos with their smart phones at the end of the lesson.

The teacher then said the class could play the 24 game. This was a game that is used with grade 2 (England Year 3) students in Shanghai to practise number skills. Jokers and picture cards are removed from a normal pack of playing cards. Students work in groups of four and turn over four cards at a time. The challenge is to use all the cards to make 24 for example 5, 5, 7, 6: $5 \times 5 - (7 - 6)$. In Shanghai, this game is developed in grade 5 to make 60 where the picture cards are assigned values of 11, 12 and 13. The students relished the challenge and worked enthusiastically together, including those with weaker number skills.

For both lessons, we were struck by the teachers' careful task design. The tasks had to be of particular types to develop mathematical skills. The Shanghai textbooks use problems to engage and motivate students. The problem: Will the rod that measures $\frac{5}{6}$ cm fit in the hole that measures $\frac{7}{8}$ cm? starts the section on comparing and ordering fractions.

We were told that models and images had been used earlier in the sequences of lessons but none were used in the observed lessons. It appears that the move to abstraction is fairly rapid in Shanghai secondary teaching.

We were conscious that in the fractions lesson we might want to emphasise mental methods as a first resort and make general links to notions of difference so that $7\frac{1}{9} - 3\frac{8}{9}$ should be thought about before moving into computation. Adding $\frac{1}{9}$ to both terms results in a much easier calculation with the same answer: $7\frac{2}{9} - 4$. Given the emphasis on developing understanding of mathematical structure which seems to be important in the Singapore textbook pilot and the NCETM's Mastery agenda, we were surprised at the emphasis on 'the method' albeit that several methods were offered in

both lessons.

It did not occur to the teacher of the mean that students might have been struggling with negative numbers. In Shanghai, children start school with a great deal of number knowledge, number bonds to 100 and multiplication facts that are a strong basis for developing mental arithmetic skills.

In discussion with the teachers it was clear that there are vast cultural differences. Some of these are:

- The belief that all children can learn and be successful, they just need to work hard.
- The extraordinarily high expectations of all children in Shanghai. Academic success is seen as key to a successful life.
- Parents and grandparents look to the child to support them in their old age. "A child doesn't have two parents, they have six, the parent and the grandparents."
- Poverty is apparent even in the highly successful Shanghai province and children know that if they do not work hard they could end up 'in the gutter'.
- The children in Shanghai are tired. When they get home from school they must complete a lot of homework as the competition is fierce.
- At weekends, they have private tuition in English, mathematics and Chinese.

One of the teachers commented that English schoolchildren are much happier than their Shanghai counterparts. They are also much better at asking questions.

Some key points arising from our visit that resonate with messages from research included: a) Giving children time to learn; b) mental arithmetic; c) offering a number of methods; d) there is no setting; e) the use of textbooks.

- a) Many researchers have found that spending a substantial amount of time on a topic actually leads to more effective learning than regular re-visiting where previous failures are reinforced (Watson, 2006). The ATM publications *Big ideas* and *Bigger ideas* exemplify this approach in schemes of work for Y7 and Y8. In contrast to the Chinese approach, which follows a carefully constructed trajectory with lots of practice, we advocate developing deep conceptual understanding through a rich experience of problem-solving type tasks. There are many sources for such tasks: Association of Teachers of Mathematics (ATM), NRIC www.nrich.maths.org, the Mathematical Association (MA), www.

inquirymaths.co.uk, Mathematics in Education and Industry (MEI) and the STEM e-library which has all the SMILE resources, Cre8ate Maths, the Standards Units and much more.

- b) The new national curriculum has lost 'mental methods as a first resort' which has been part of the mathematics programme of study since its inception. We do not consider rapid recall of number facts to be at the heart of mental arithmetic, rather an "at homeness with number" which allows students to flexibly decompose and recompose number as befits their confidence in the context (Ollerton, 2014).
- c) The Shanghai teachers placed a great deal of emphasis on the use of a number of different methods. These methods were explicitly taught and their suitability in different contexts was discussed. This is in stark contrast to the new national curriculum for primary mathematics where specific methods of calculation are prescribed and will be tested. Of course, children will get full marks for correct answers, however they are arrived at, but many primary teachers feel under pressure to train children in the prescribed methods. Using a variety of methods can help the development of conceptual understanding and make connections in mathematics explicit.
- d) In Shanghai, as in many countries around the world, children are not placed in sets for mathematics. OECD (2009: 6) is clear that in order to improve outcomes for all learners then "limit early tracking and streaming and postpone academic selection". Boaler (2009) is a well-known advocate for all attainment grouping in mathematics with substantial research evidence of the benefits. The ATM All Attainment working group regularly explores possibilities.
- e) The Shanghai textbooks we saw were carefully crafted, drawing on teachers' experiences to refine them over time. The textbooks match the curriculum that is reviewed every ten years, in contrast to the frequent changes of the English national curriculum, since its introduction in 1988. We know it is feasible to 'teach' the whole of the statutory mathematics curriculum without needing to use a textbook, there is an ATM publication *Learning and teaching mathematics without a textbook* (Ollerton, 2002) that provides support for anyone who is interested. Of course, arriving at such a position is achieved through experience and collaborative construction of schemes of work. Learning to teach does not happen overnight and neither does it happen because a government prescribes specific methods

(remember the National Strategies).

There were two important aspects with which we felt less comfortable in terms of how they might impact upon mathematics teaching in England:

a) the use of practical resources; b) differentiation

- a) ATM began life as the Association for Teaching Aids in Mathematics. The use of practical resources, manipulatives and models can help students to make sense of mathematics. These aids are not 'the mathematics' rather they support students to develop mathematical understanding. Whatever stage of mathematics you are working at the use of resources and images is essential, even as a research mathematician (Burton, 2001). The use of models and manipulatives supports being able to move confidently between equivalent representations (for example numerical, graphical, symbolic). Of course, we only saw two lessons so we are in no position to generalise. However, in discussions with the Shanghai teachers it was clear the lessons they taught did not elevate or give priority to the use of practical resources.
- b) As previously mentioned, when asked about how the Shanghai teachers supported those learners who were clearly struggling with the pace of the lesson and those for whom the lesson was not sufficiently challenging, we were given the following explanations:
- Shanghai mathematics teachers do not teach the first two hours of their school day, which starts at 07:30h. In this time, they collect and mark their students' homework so it can be returned to them in their next mathematics lesson.
 - At the end of formal lessons, teachers are available to support those students who struggled in the lesson or need extension work. This is the students' responsibility and is utilised accordingly, possibly because of the familial high expectations placed upon each child.

Clearly, there are significantly different cultural expectations between Shanghai-educated and English-educated children. This is not intended to be any kind of excuse as to why the former are purported to be stronger mathematicians, however, seeking to transpose specific aspects of the ways Shanghai children are taught and how they learn into English schools is fraught with complexities.

Furthermore, English teachers teach for most of the school day by contrast to Shanghai mathematics

teachers who teach only two or three 40-minute lessons each day. Likewise, English children do not spend up to 12 hours of the day in school, several hours doing homework and their weekends with private tutors. We strongly advocate, however, that the vast majority of the students should have access to the statutory curriculum. The Shanghai model of 'teaching to the middle' in teacher-led lessons seems unlikely to inspire and motivate all students to become confident and successful mathematicians eager to learn more. Offering all students the opportunity to learn more of the curriculum to ever greater depths and believing they can do so may well help to improve the lot of English students. However, learning is highly personal and it is through knowing our students that we are most likely to be able to support them through the challenges of learning mathematics, building their confidence, resilience and enjoyment of mathematics. Differentiation is an essential and inevitable element of all teaching (Ollerton, 2014).

To conclude we offer a quote by Bruner:

... any subject can be taught to any child at any age in some form that is both honest and powerful. It is a premise that rests on the fact that more complex abstract ideas can in fact be rendered in an intuitive, operational form that comes within reach of any learner (1972, p122)

It seems to us that only something that takes account of culture and context can be honest and powerful. We have rarely met teachers who are not trying to do their best for their students. We look forward to a time when, like in Shanghai:

- Children do not begin formal mathematics until the age of six.
- The vast majority have access to the entire curriculum and are not labelled from an early age as to whether or not they can learn mathematics.
- Teachers are expected and enabled to engage with high quality professional development.
- Politicians do not change the curriculum and assessment on a whim, but every ten years drawing on the experience and expertise of a respected teaching profession and academics.

Overall, we found our visit raised key issues about how children learn and how teachers set out to support and develop such learning. We hope to have presented a balanced piece of writing regarding these issues that, after all, can only begin to make sense within the different contexts in which Shanghai and English teachers and their schools exist.

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Mike Ollerton is passionate about mathematics teaching, problem solving, mixed-ability, fell walking, cycling, LFC, bridge, dearly beloved (db), not necessarily in that order.

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ATM Conference 2016 Mathematics as a Human Endeavour

This year's Easter Conference will be held in Warwick at Chesford Grange, 29 March to 1 April

Our attention as teachers is so often focused on mathematical content, on symbols and representations, on tasks and on schemes of work, that we may lose sight of our subject as one fashioned by people. Mathematics is invented, often to respond to the computational demands of the age such as the 16th century developments in astronomy, navigation and map making. The creation of logarithms facilitated multiplication by turning it into addition. In the 19th century, in the course of his astronomical work, Gauss invented a notation to solve the equations associated with the orbit of asteroids. From Gauss' innovatory work later mathematicians devised what we now call Gaussian Elimination as a means of solving simultaneous equations.

This year's Easter Conference theme is intended to redress an over-emphasis on mathematical outcomes and emphasise the process of arriving at them, a focus on the doing not just the done. As usual the theme of this ATM Conference will be addressed directly in some sessions and not at all explicitly in others, but the conference theme reflects the abiding ATM belief in mathematics as something done by people, and what that means for teaching and learning.

Conference sessions are accordingly biased towards activity, discussion and practical tasks echoing ATM emphasis upon *doing*, on mathematics as activity, on making conjectures and testing them, on deriving results and generalising from them. A glance at the programme on the ATM website

(<http://www.atm.org.uk/Maths-teacher-conference>) reveals the range of classroom-relevant sessions, partnered by sessions that look behind the immediacy of the classroom to empower reflection to support CPD needs. The emphasis on mathematics as activity is currently particularly pertinent: the new GCSE and forthcoming revised A Levels require learners to solve problems, not as at present only to use practised techniques on stereotyped examination questions.

We open the conference with Jackie Fairchild's plenary titled *Capturing Mathematical Relationships through Representation: a Timeless Human Endeavour*. The conference ends with David Cain's closing plenary, *Our Mathematical Endeavour – Creative or Non-Creative?*

We look forward to meeting you all, new friends and old at Chesford Grange on 29 March.

