In conversation

Mike Ollerton, Clare Benton, and Jenni Back discuss assessment

Mike: With the new national curriculum for mathematics and its aims of fluency, reasoning, and problem solving, I have been thinking about how teachers might set about assessing learners' achievements against these aims.

I wondered how you seek to assess your learners in your school. Let's focus on problem solving.

Clare: I think there are two separate aspects. One is how to assess students when they are in a lesson and working on a mathematics problem, and the other is the more formal assessment when we assess students' responses to a written problem in a test situation. Unless students have developed the skills, mathematical thinking and resilience, required to solve problems in their lessons; many will struggle to apply their mathematics knowledge in a test.

There is a difference between working in a lesson where I am seeing and listening to their responses and assessing them informally to determine my next steps in teaching. This contrasts with formal test assessments where you only have the written response as evidence of their problem solving skills. I am aware I need to work on how to develop students' habits of working so the formal context offers them no more challenge than problem solving in the classroom.

Here is an example of a question my Year 9 group answered poorly in their recent test on ratio.

21 Here are the instructions on a bottle of fruit squash.

To make fizzy juice
mix 2 parts fruit squash
with 7 parts lemonade

21 (a) How much fruit squash is needed to make 450ml of fizzy juice?

[2 marks]

Answer _____ ml

Figure 1

21 (b) Tom has 80ml of fruit squash.

He also has 210ml lemonade.

What is the **maximum** amount of fizzy juice he can make?

[3 marks]

Answer _____ ml

Figure 2

We had spent a long time on ratio in different contexts, so it was disappointing that the common answer was 290ml (which they got by adding together 80ml and 210ml). We did a follow up lesson on this to explore where the difficulty was for the girls and how they approached the question. One student replied '*I was going to multiply by 40 but that would not have given the correct ratio'* which implied the confusion was in their understanding of what the question was asking. It also highlighted they haven't yet developed the skills to check their understanding and to show that the quantities (80ml and 210ml) weren't given in the correct ratio (2:7).

Jenni: This is really interesting because it problematises the issue of assessing understanding using a timed-written test. What I am interested in, now, is what you do in the classroom and how that addresses the issue of problem solving.

Clare: My concern about how the formal (end-of-year) assessments are not reflecting students' understanding has led me to look to include more reflection by students on their work. We intend to address this through using student journals, where they reflect upon and explain what they have understood from a topic and/or the 'best' problems they have solved. In the meantime we have been collecting reflections. An example is shown in Figure 3.

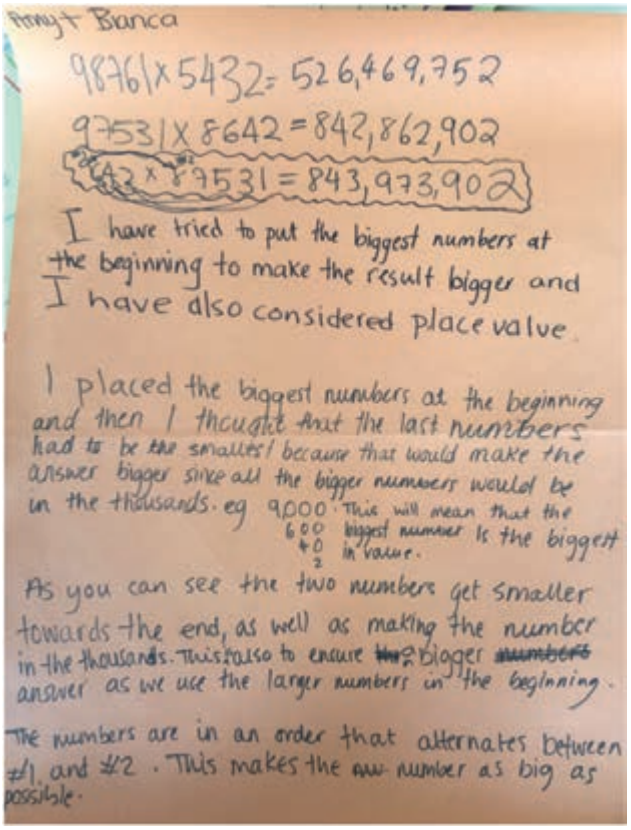


Figure 3

In this example I was interested in what their thinking was behind their choice of digits to make the biggest possible product of two numbers including all the digits 1 to 9 just once each. Because I had been talking to Mike about problem solving, I had asked students to explain their thinking every time they made a change in their strategy, or approach, to the task. This proved a distraction from their *in the moment thinking* about the task which they didn't value. We got round this by asking students to give a post-hoc account at the end of the lesson and the example shows how two girls did this.

More recently I have been allowing time for independent individual reflection at the end of the lesson in order to help students articulate descriptions of the mathematics they have used and their approaches to the tasks. I believe this will become useful in assessment contexts, as they become more able to identify consciously the mathematics involved and their approach to it. This should help students to see their own progress as mathematicians, and the next steps in their learning.

Jenni: This particular example shows three key aspects of problem solving:

- using and applying mathematics
- having a deep understanding of the required mathematics
- being able to draw on an appropriate range of problem solving skills

The girls show evidence of applying their knowledge of numbers and the number system in their work; they demonstrate a deep understanding of place-value and the notion that the further 'to the left' the digits are placed, the bigger the value of the digit; and they work systematically through the possibilities to identify the best solution.

Mike: I also see evidence of communication skills by Bianca and Amy and communication was one of the criteria incorporated into the ATM-SEG GCSE which is written up in MT203. This involved students being *"able to present methods and results in an orderly sequence" and "taking opportunities to use mathematical symbolisation…"*

Such communication skills are, I believe, an essential component of problem solving; to make the implicit explicit.

Clare: I worked hard at the beginning of the year to gain their trust so that they are prepared to engage with me in this kind of classroom endeavour. Students come from backgrounds that rate exam performance highly, but do not necessarily understand how excellent results can be achieved. At the beginning of the year, they would ask what to do next, and wanted to know how relevant, or useful, the task was. Now they trust me to offer opportunities that will support their learning and they seem to feel empowered to tackle mathematics problems.

Jenni: Can you say a little more about the difference between the formal, end of year assessment, your ongoing assessment for learning and, what you refer to as the learning assessment?

Mike: Yes. There are some key issues here about:

a) how exam/test questions are written;

b) whether tests can have a positive impact upon how mathematics is taught and

c) what alternatives there might be.

An email you sent me earlier this year, Clare, contained a couple of your test papers.

Clare: Here are two examples of 'learning assessments we use at Bentley Wood. The first is about percentages. See Figures 4 and 5.

Learning Assessment 1 - Year 9 Figure 4

Topic: Percentages

Questions.

1. Talk me through how you would increase £12 by 15%.
   Can you do it in a different way?

2. The answer to a percentage increase question is £10.

   Make up an easy question.

   Make up a difficult question.

3. 3000×0.83^3
   Does the multiplier in this calculation result in an increase or a decrease? How do you know?

4. How is compound interest different from simple interest?

Teacher Comment

Pupil Response

And the second Pythagoras' theorem:

Learning Assessment 2 - Year 9 Figure 5

Topic: Pythagoras' Theorem

Questions.

1. How do you identify the hypotenuse when solving a problem using Pythagoras' theorem?

2. What do you look for in a problem to decide whether it can be solved using Pythagoras' theorem?

3. A 5m ladder leans against a wall with its foot 1.5m away from the wall. Hannah says the ladder reaches 5.2m up the wall. Is she correct?

4. In a rectangle measuring 12cm by 8cm, is the hypotenuse 15cm long?

5. Is this triangle right-angled? How can you tell?

Clare *(continued)*: I think that's why I was surprised by the students' responses because the assessments we had done throughout the year were of this nature but sadly they hadn't yet had an impact on the end of year exam.

Jenni: These are really interesting. They capture the essence of what I would hope students would demonstrate understand of after studying the topic.

- The questions are provocative and demand problem solving skills.
- Students wouldn't be able to just trot out a rehearsed response.
- Students are being asked to construct their own questions which is known to be valuable (Standards Unit box)

Can you tell us a bit more about what you do with these assessments?

Clare: They are done at the end of the topic, in the classroom, and the students are free to ask me questions about their responses if they want to. Once they are marked, we use them to inform us about areas we need to address before moving on.

Jenni: This is assessment for learning as I think it should be; you are using the students' responses to inform your teaching and address their misunderstandings and gaps in their knowledge. This assessment is being done for a clear purpose and will be of direct value to the students - never mind data gathering!

Mike: This all sounds incredibly, pedagogically sound. If learners come to expect these kinds of questions in their mathematics lessons, then it would seem useful for such questions to appear on written tests. However, I am now interested in the disjoint between the formal end-of-topic assessment and the ongoing learning assessments for the ratio and proportion topic.

Clare: We had worked on a variety of problems involving making lemonade and mixing paint.

Fluency won't happen without practise, but over use of practise questions results in students not thinking about when to apply knowledge or a concept. There a few challenges here – finding good practise questions balanced with the time to allow for problem solving. Fluency is what we are aiming for however, balancing problem solving with practise questions is a challenge if we are to show students making progress in end of year exams.

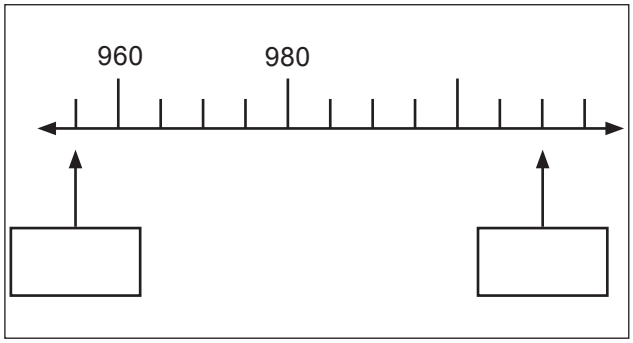
Jenni: This all resonates very strongly in relation to my work with primary school teachers and children. I have been developing a scheme of work with a school in Brighton to build on a similar approach to problem solving as one of the main aims of all the mathematical work the children undertake. We introduce every new topic with a problem solving activity, which we describe as a 'hook'. For example, for number sense and place value with Key Stage 2 students we use dice games where you choose where to place the number thrown on a grid (see for example Nice and Nasty https://nrich.maths.org/6605).
This offers a context in which the children can see what mathematics they need to know in order to solve the problem.

We use mind maps as an age-appropriate alternative to journal writing. We give children the lists of relevant words and concepts to support them in creating their own, or a class, mind map, describing what they know already. That mind map is then revisited at the end of the topic and the children adapt it to include what they know now. See below for an example created by a Year 5 class on the topic of number sense and place value with their teacher acting as scribe for the children's suggestions:

Figure 6

Assessment is also undertaken more formally by using practise assessment questions from collections of such tasks.

The task below asks the child to identify the number indicated by the arrows:



each topic has a linked collection of applications tasks in which linked measures and word problems are tackled.

The children are actively encouraged to think about what they have learnt in revisiting and revising their mind maps, and also in creating '*memory joggers'* to keep in their '*mathematics packs'*. The final celebration at the end of each topic will include a sharing event with another class, parents and carers, or an exhibition of their work. We hope that all of this will help support the children's responses to the assessments that they face.

Conclusion

We believe the most productive types of assessments are those which are constructively aligned to everyday teaching and learning (Biggs 2003). Finding ways of integrating assessment into schemes of work, into lesson planning and into learning, and recognising the value of stimulating learners to reflect upon and communicate their mathematics are powerful ways of supporting learning.

Mike Ollerton is an Independent Consultant, Clare Benton teaches at Bentley Wood High School, Jenni Back is employed by NCETM

References

Biggs, J., (2003): *Aligning Teaching and Assessment to Curriculum Objectives,* (Imaginative Curriculum Project, LTSN Generic Centre)

Ollerton, M., and Watson, A., (2007) GCSE coursework in mathematics, *MT203,* Association of Teachers of Mathematics, Derby

New Editor

As from January 2016 there will be a new editor. He is Tony Cotton, currently a member of GC. All articles should be sent to him via the email journaleditor@atm.org.uk