DIFFERENTIATION IN MATHEMATICS CLASSROOMS

Mike Ollerton discusses the wider implications of differentiation in the mathematics classroom

Learning is messy. It does not occur in neat linear steps; it is a complex process. Sometimes we miss seeing the most obvious idea, whilst at other times we make intuitive leaps of understanding, often when we least expect it. My favourite example was waking in the middle of the night when studying an Open University mathematics degree course and finding I had worked out the solution to a problem involving the most complex piece of mathematics I had ever engaged with then, or since.

In A-level mathematics the concept of differentiation $^{\delta y}/_{\delta x}$ is basically about rate of change based upon infinitesimally small changes to a function; to the slope of a curve. This is a useful analogy to consider with regard to differentiated learning; similarly students learn at different rates according to all kinds of small, often imperceptible changes. This happens both because, and irrespective of, what teachers do. This is not an attempt to write the teacher out of the equation, far from it; the quality of stimuli teachers' offer students impacts upon the quality of thinking and the depth of sense-making different students achieve.

Differentiated learning happens; full stop. *It's impossible not to achieve differentiation in the classroom, no matter what you as a teacher decide to do*. Brown (1995, p33). Because differentiation occurs irrespective of what we do this means even when asking a closed question such as: "*What is 23 plus 19*?" Although this has a single answer, students will think about the answer at different processing speeds and have different degrees of confidence to offer their answer. I am not for a moment advocating the value of asking such a closed question to a whole class, merely using this as an example of the inescapability of differentiation. Of course we can also turn this question around and ask more open versions such as

i) *Discuss with a partner how many different ways you can think of finding the answer to 23 + 19, or*

ii) *"If the answer is 42 what might be the question?"*

Indeed when I asked this question to a class of 7 to 8 year old learners I was delighted one of them answered: "*The answer to life, the universe and everything*"; going on to explain that his Dad was reading the *Hitch Hikers Guide to the Galaxy* to him. Returning to the question, there are of course different answers and these, in turn, create opportunities for different students to provide different answers offering different degrees of complexity.

Furthermore, differentiation does not happen at some spurious notion of three different levels; it happens at as many different levels of cognition and depth of sense-making as there are students in a class. Differentiation, therefore, is probably the most complex and important issue for teachers to engage with. It is omnipresent. How we embrace differentiated learning in our planning-for-teaching, and in our interactions with students are crucial considerations.

Because in any class, as suggested above, there will be as many different levels of engagement, confidence, speeds and depths of understanding as there will be students in the class. A fundamentally important issue, therefore, is how to plan for the inevitable differentiation that exists?

Planning for the inevitability of differentiated outcomes

The following quotation from a seminal publication *Mathematics from 5 to 16* is unequivocal about the importance of planning: *Differentiation of content, if well planned, facilitates progression for all pupils*. DES (1985, p26). The quality of the tasks we provide/offer/set up in terms of their initial accessibility and potential extendibility cannot be understated. Seeking to offer tasks which are intended to provoke active engagement, because they are accessible and sufficiently interesting for students to engage with in the first instance is imperative. Planning the different depths a task might be taken to, whilst recognising that some of the more powerful tasks are those which students extend themselves, is a further key criterion. As an example I offer the *Sums and Products* task, which I first met in an Association of Teachers of Mathematics (ATM) publication, *Points of Departure*, (book 3, idea 23) which works as follows:

Choose a number and ask each student to write four or five partitions of that number.

Several students could be asked to provide one of their answers and these could be written on the board/screen. For example, using the number 10 students might offer:

5 + 5

2 + 3 + 5

4 + 5 + 1

6 + 4

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1

This would clearly be accessible to the vast majority of students more than seven years old.

The next part of the task is to turn each addition sign into a multiplication sign, and for learners to calculate the resulting products gained, i.e.

5 x 5 = 25

2 x 3 x 5 = 30

4 x 5 x 1 = 20

6 x 4 = 24

1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 = 1

The intention now is for students to see the different range of products possible. One aspect of teacher questioning might be to ask students to produce the partitions that gave rise to the answers provided by asking other students. For example, if someone offers an answer of 32 the teacher could ask students to consider what the calculation was. The value of this is to cause students to work in reverse.

The main point of this task is for students to explore which partition produces the maximum product - which will be 36, where the calculation will be 3 x 3 x 4, or 3 x 2 x 3 x 2. As this result is achieved the task develops by asking students to choose their own starting number and repeat the above process, again looking how to maximise the product.

There will be opportunities for the teacher to discuss with students the use of exponents to write some of the calculations in mathematical shorthand.

Thus 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 x 1 can be written as 1^{10},

likewise 2 x 2 x 2 x 2 x 2 as 2^5.

Posing the problem were students look for a way of being able to predict how to create the maximum products for any number will be a substantial undertaking, especially as '*the answer*' lies within the domain of modular arithmetic and exponents.

Another key quotation from HMI mathematics from 5 to16 engages with the intrinsic value of extension tasks as a way of supporting differentiated learning: *Mathematical content needs to be differentiated to match the abilities of the pupils, but according to the principle quoted from the Cockcroft report, this is achieved at each stage by extensions rather than deletions.* DES (1985, p 26). Thus, with regard to the task above, different extension tasks can be worked on, for example:

i) What happens if the rule for partitioning into non-integer values is allowed.
What will the maximum product be under this circumstance?
For example 2.5 x 2.5 x 2.5 x 2.5 = $(2.5)^4$ and $(2.5)^4$ = 39.0625; which is clearly bigger than the previous product using integer values, but is this the largest possible answer.

ii) Another avenue to explore might be to consider what happens if we only use values using just one partition for example,5 + 5, 3 + 7, 9 + 1 etc.
Here we achieve products of 25, 21, and 9.
However, students could be asked to consider what the graphs look like when the following co-ordinate pairs are plotted: (5, 5), (3, 7), and (9, 1) – clearly these would sit on the line $x + y = 10$.

iii) A more complex graph would be to consider just one of the partitions against the product for example, (5, 25), (3, 21), (7, 21), (9, 9), and (1, 9). This will produce a negative quadratic graph which has the equation $y = x(10 – x)$.

There are clearly different extension tasks which different students can develop to different degrees of complexity; thus possibilities for differentiation abound.

This task is problem-solving by nature. Another description which has caught hold in the last few years is that of a *rich mathematical task*. This notion however was first defined in *Better Mathematics* (1987, p20) at an inset course by a group of teachers as:

- *must be accessible to everyone at the start;*
- *needs to allow for further challenges and be extendable;*
- *should invite children to make decisions;*
- *should involve children in speculating, hypothesis making and testing, proving or explaining, reflecting, interpreting;*
- *should not restrict pupils from searching in other directions;*
- *should promote discussion and communication*
- *should encourage originality / invention;*
- *should encourage 'what if' and 'what if not' questions;*
- *should have an element of surprise;*
- *should be enjoyable.*

Each of these elements describes quality teaching, which, in turn, automatically embraces differentiated learning. In the same publication, p15, there is another interesting, entertaining yet pedagogically sound list which compares '*Junk Food*' and '*Junk mathematics*'!

Differentiation and whole class discussion

The essential issue of differentiation with regard to whole class discussion is about the depth of understanding any student takes from the discussion.

Questioning strategies which do not require a traditional 'hands-up' approach can be powerful in terms of keeping everyone on their toes, and for creating the expectation/culture that everyone has something to offer. The 'think, pair, share' strategy which, I believe, is grounded in effective practice has been used in some primary schools for very many years and is beginning to be utilised in secondary and in tertiary education. There are other approaches such as the use of a random name generator, the 'card' trick, the 'two dice' idea. The card trick works as follows:

The teacher takes a pack of cards and shows a different card to each student as they enter the room, possibly asking them to write their card in the back of their books or their mini-whiteboards. The teacher offers students a stimulus, such as a picture, a graph, a random collection of numbers, a shape etc then turns over the first card and that student is expected to say something about the stimulus.

I have seen this 'card' approach used most effectively in a Year 8 class; the two dice approach worked superbly in another class where the teacher had organised the classroom into six work 'stations'; students knew their table number and the number on the first die identified the table number. Each student then had their own number for their table and this was identified by the number on the second die; so 5, 2 was table 5 student 2. Thus the roll of the die determined who was going to speak next, but this gave the teacher a great opportunity; having identified who is to speak then next to decide what kind of question to ask. Perhaps stronger students might be asked to justify the previous answer, perhaps another student could be asked to '*give another example'* of what a previous student had offered. I saw both of these strategies used to great effect in a Year 8 and a Year 7 class, and I felt both approaches strongly supported learning.

Whilst both approaches undoubtedly worked well this does not mean these, or any other approaches have to be rigidly adhered to. For example, if during the period of time when students are engaging with a task the teacher has interesting exchanges with some of the students, then an added approach could be for the teacher to invite the student(s) to explain to everyone else what they had told their teacher. Again we are working with differentiated outcomes.

Differentiation and the 'Mantle of the expert'/'flipped' classroom

I have noticed recently on Twitter the concept of '*Mantle of the expert'* has begun to appear, though it is a phrase constructed many years ago by Dorothy Heathcote (1926 – 2011) who was a Drama in Education teacher. Dorothy worked for many years with all kinds of organisations for example, British Gas, and taught on PGCE and Masters Level courses at the University of Newcastle-upon-Tyne. I had the good fortune to meet her in 1983. Anyway, "*Mantle of the expert*" as it suggests is about conferring on learners a *mantle of expertise*. This meant that learners engaged in research, discussion and sharing in order to gain the knowledge for which they had been designated experts of. Over the past couple of years, to my knowledge, the construct of a '*flipped classroom'* has emerged in educational parlance. This is where learners are given responsibility to teach something to the rest of their class, having previously been tasked to do so by the teacher. This means the student(s) have to prepare something in advance of 'the flipped lesson'; a strategy firmly grounded in '*Mantle of the expert'*. In turn, the flipped classroom is just one strategy which sits comfortably within the wider construct of enquiry-based learning (EBL).

The '*flipped classroom'* has taken on an increased significance over the past two or three years and is in part associated with the increasing amount of information available on the internet, which includes sites such as the Khan academy:
www.khanacademy.org
One intention is to develop students' personal qualities, such as independence, and responsibility, for their own learning.

Recently, in September and October 2013, I was invited to work with a couple of classes on the topic of percentages. I had no idea what students already knew, or who the least and most confident students were, or in fact anything else other than they were 12 to13 years old. I decided, therefore, to employ my own version of a flipped classroom, and to do this I gave students the information sheets which appear in Appendix 1. The intention was for students to choose: with whom they wished to work in a group of 3, and which problem to work on. I explained that problem 1 was the easiest, and that problem 6 was the most challenging.

Differentiated learning occurred both in terms of the task students chose to work on and the outcomes achieved. At one point in the first lesson I noticed two of the groups had chosen problem 6, and I made a professional judgement that both groups would have benefited from my input with regard to developing the task. To achieve this I asked all six students to join me in a space in the classroom and asked them if they could think of a way of shortening their calculation of continuously multiplying by 1.1, in order to achieve each 10% increase. Quite quickly one student asked if they could use exponents. I asked the student to explain to the others what he meant, and it was clear that the groups were able to return to their tasks with this enhanced knowledge. The whole intervention took less than two minutes. In Appendix 2 are some further tasks which I indicated would be tasks for later lessons. Task c) of Appendix 2 is in fact a subsequent addition, which I did not offer, but would do if I were to use the idea again in the future.

Note: All appendices to this article can be found online at www.atm.org.uk/mt240 or by clicking on the red QR code at the end of the article.

Quality planning, quality teaching

Returning to the issue of quality, teachers who plan for the inevitable differentiation recognise the impact the stimuli they offer has upon student learning. What follows might read like a 'formula' for successful teaching, however, I base my thinking upon firstly my learning as a consequence of when a teacher has enabled me to shine and, secondly, my teaching when I believe I have enabled students to shine.

The conditions upon which I believe quality teaching is based are:

- Planning which begins with a question such as: "*How can I cause my students to get actively involved with and engaged in…?*"
- Stimuli, which are initially accessible and easily extendable by students or the teacher.
- Teaching strategies based upon more open-ended questions and stimuli where students, perhaps working in pairs, are given a couple of minutes to discuss their response to such questions.
- Problem-solving approaches which enable students to deepen their thinking.
- Projects which students can develop over time and see for themselves the progress they make.
- Planning which causes students to go right back to their basics in order to remind them of what they need to know to move forward. Of course 'basics' is a changeable feast; basics for EYFS students will be about counting, number recognition, and shape recognition. For Year 12 students some basics will be their knowledge of sequences, functions and graphs, which will enable them to engage with, and make sense of, calculus.

At the beginning of this piece I used the analogy of mathematical differentiation as a metaphor for learning at different rates of change. I cannot miss this opportunity to complete the piece by using the idea of integration within calculus, as a metaphor for inclusion. Mathematical integration is about the area under a curve; educational inclusion, meanwhile, is about including everyone in a class in learning whatever concept is under scrutiny. The following quotation from Bruner (1972, p122) comes to mind:

With respect to making accessible the deep structure of any given discipline, I think the rule holds that any subject can be taught to any child at any age in some form that is both honest and powerful. It is a premise that rests on the fact that more complex abstract ideas can in fact be rendered in an intuitive, operational form that comes within reach of any learner to aid him towards the more abstract idea yet to be mastered.

Bruner throws down a challenge which I believe we rise to by seeking to include everyone. Inclusion is fundamentally about how we engage in our planning-for-teaching and our moment-by-moment interactions in classrooms, constantly taking the key issue of differentiation into account.

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References

Ahmed, A, (1987), *Better Mathematics: a curriculum development study,* London, HMSO.

ATM (1989) *Points of departure,* Derby, Association of Teachers of Mathematics.

Brown, L. (1995) *What would it be like not to differentiate?*, Mathematics Teaching 152, Derby, Association of Teachers of Mathematics.

Bruner, J. S. (1972) *The relevance of education,* Trowbridge: Redwood Press Ltd.

DES (1985) *Mathematics from 5 to 16,* HMI Series Curriculum Matters 3 London, HMSO.