‘FLUFFY’ ASSESSMENT, JOINED-UP THINKING, VISION AND FUTURE ASPIRATION

Mike Ollerton and Daniela Vasile explain how notions of appropriate forms of assessment can challenge both learners and teachers

This article explores some issues and complexities of carrying out assessment of students’ achievements which take into account their capabilities to explore, to use and apply, and to communicate their understanding of mathematics, beyond the confines of narrow, ‘traditional’ testing. The piece discusses pedagogic issues and seeks to contextualise and exemplify them with events involving ‘*real*’ students in ‘*real*’ classrooms.

“*Venture yourselves into a tessellated world where this time you will be in jeopardy. When the Question ‘Titan invented’ a question of impossibility, you will have to figure out how to solve it. Prepare for endless possibilities.*”

This is how Adrian, a Year 7 student at South Island School, Hong Kong, starts, and perceives, his assessment on *Regular and Semi-regular Tessellation*. The assessment question asked was: *“There are only three regular tessellations. TRUE or FALSE?”* This event, however, is somewhat out of sync with a sequence of earlier events from 2010 when Mike Ollerton began working with the mathematics department at South Island School (SIS), Hong Kong, where Daniela Vasile was, and continues, to be head of the mathematics department. Mike had been asked to develop ‘*enquiry-based learning*’ (EBL) in the department. In the main this involved the department changing from traditional, textbook orientated teaching and “*venturing*” into the world of mathematical enquiry; of problem solving and encouraging more independent learning.

From October 2010 to April 2012 Mike had three separate, intensive weeks with the SIS mathematics department and shared hundreds of email conversations with Daniela to iron out difficulties and to discuss wider implications of adopting EBL. In October 2012, Daniela described a concern a colleague had expressed about giving students a ‘*proper*’ assessment, about angles and regular polygons, rather than a ‘*fluffy*’ assessment, which started with the question: *“Which regular polygons tessellate?”* Mike was interested in this on three counts and we wondered if these were worth sharing with a wider audience?; in part because we sense the issues of assessment, for the SIS mathematics department, of the processes involved in working mathematically may be similar for other mathematics departments.

The first issue is about how ‘*proper*’ assessments and ‘*fluffy*’ (EBL) assessments are defined. Our concern is how the former seems to suggest a form of assessment based upon some notion of rigour, of giving pupils a test using questions which have only ‘right’ or’ wrong’ answers in order for marks to be awarded leading to a percentage score. The latter, however, challenges such a clear/precise methodology as learners’ answers might lead into grey areas of answers being partially correct; thus being able to allocate marks might create ambiguities. Importantly an EBL form of assessment enables the teacher to see the kind of conceptual connections students form between different areas of mathematics, i.e. angle, tessellation, and factors. This as an important part of learning mathematics, as recognised by Ofsted: *Pupils understand important concepts and are able to make connections within mathematics.* (Ofsted ‘Outstanding’ criterion 2013)

Secondly, if ‘traditional’ testing is seen as a useful way of assessing pupils’ knowledge, of their ability to recall facts and to provide answers to many questions in a short space of time, how and when are important skills attributed to problem solving/mathematical thinking going to be assessed? In brief, how is it possible to assess students’ capabilities to engage in mathematical thinking, to use and apply processes such as:

- Collecting
- Sorting/grouping
- Ordering/systematising
- Exploring
- Measuring
- Comparing
- Pattern spotting
- Looking for connections
- Conjecturing
- Predicting
- Checking
- Justifying
- Generalising
- Proving
- Initiating further investigation
- Communicating/Writing about/Explaining/Presenting

Furthermore, how do we find out what goes on inside learners’ heads when they are asked to engage with the concept of angles in regular polygons, or any other concept? An alternative could be to provide tasks which encourage pupils to explain:

- How to draw the graph of a function
- What it means to solve two equations simultaneously
- What a semi-regular tessellation is
- How to convert a fraction into a decimal
- What 2.7 means
- What $\sin x$ means

In turn, such tasks would provide students with valuable opportunities to communicate the mathematics underlying such concepts and at the same time provide teachers with valuable information about the students’ deeper understanding.

Of course there is a plethora of even deeper "why" type questions, which search into the core of a students' understanding. For example:

- Why does $a^2 + b^2 = c^2$? (Where a, b and c are the lengths of the sides of a right-angled triangle)
- Why do the graphs of the form $y = mx + c$ cross the y- axis at (0, c)?
- Why do the angles of a triangle sum to 180°?
- Why does a square and a regular octagon tessellate?
- Why do the dihedral angles of a tetrahedron and an octahedron sum to 180°?
- Why is the sum of three consecutive numbers a multiple of 3 but the sum of four consecutive numbers is not a multiple of 4?

This leads us to a third concern which is about separating assessment from everyday classroom practice; making assessment a bolt-on afterthought rather than an integrated event in everyday teaching and learning. Of course this is the difference between summative and formative assessment, we question whether the kind of assessments carried out under the title 'summative', really do summate students' knowledge; let alone their problem-solving capabilities.

For the mathematics department at the South Island School, this meant the previous, 'traditional' exam-style assessments did not fit with an EBL style of teaching. We realised staff could not expect students to work in an EBL type way in a day-by-day environment and be assessed on a completely different basis, so Daniela and her colleagues decided to have a mixture of the two styles of assessment. Just as many mathematics departments in schools in the UK, the SIS mathematics department is expected to regularly, four times a year, provide scores and grades on student developments.

There is of course a massive contrast between how departments and schools are 'measured' and how they might wish to seek to equip students with capabilities to solve problems, to use and apply knowledge, and to communicate the processes which underpin mathematics. At issue is whether students will be ill-equipped to pass external examinations if they have spent time problem solving, using and applying, and communicating their mathematical knowledge.

Again there is much encouragement from Ofsted in their 2013 criteria for teaching mathematics, another of which reads: *Problem solving, discussion and investigation are seen as integral to learning mathematics*. If problem solving is to be integral to teaching and learning it must also be integral to assessment, both formative and summative.

These new approaches to learning, together with the International Baccalaureate Organisation (IBO) which SIS follows for Y12 and Y13 students and which require them to develop mathematical communication skills and the ability to use and apply major mathematical processes, which in turn require a new type of assessment. Consequently unexpected concerns arose in the department. These concerns are framed by two further issues:

1. Can we, the eight SIS teachers teaching the cohort of 220 Year 7 students, use a different mode of assessment to mark consistently and use the results of such assessments as the basis for the school report?

We sought to engage with this concern by:

a) collaboratively creating common assessment criteria, and

b) moderating a sample of students' work.

The assessment criteria SIS created were based upon three strands: Communication, Mathematical Understanding, and Structure & Organisation. Marks were awarded and weighted in the ratio of 4:5:3. In brief, teachers were looking for:

Communication - fluency, clarity, and use of mathematical vocabulary

Mathematical Understanding - collecting and sorting information, identifying patterns, seeking connections, and analysis

Structure & Organisation - use of different forms of representations geometric/algebraic/graphical/statistical, and creativity of the presentation

After all the marks were entered in the 'system' Daniela produced class-based data which showed that all classes achieved consistent results in terms of the class mean and standard deviation. Interestingly, the class ranking when compared to marks gained from a previous standard examination; this indicated that students need to be assessed using a wider range of tools in order to be given the opportunity to demonstrate their learning which takes into account their abilities to explore, to use and apply, and to communicate mathematics.

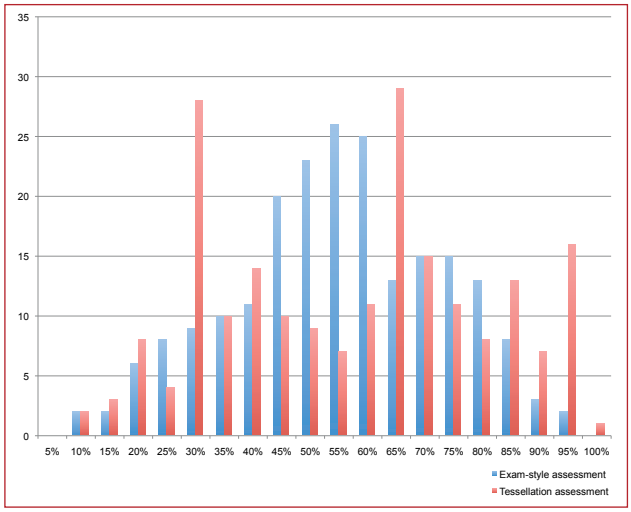
2. Is the feedback we give parents, using this different mode of assessment, valid?

This is an important question, as we live in a world were students are externally assessed in a particular way. Students need to pass IGCSE, A-Levels, and IB examinations in order to access higher education. Furthermore, in a highly-competitive environment such as Hong Kong, in parents' eyes the official examination results are *what matters*; this is the basis upon which students and teachers are judged. If our new form of assessment shows a change in the ranking compared with a traditional assessment, then would we be misreporting what a student might achieve in a '*real*' examination?

As Year 7 students are some years away from their first IGCSE examination we must trust in the long term the benefits of an assessment; and that benefits will be seen, and reflected, in any type of examination; after all mathematics is the same and mathematical processes are common, no-matter what examination board students take. Ultimately Daniela and her colleagues want to prepare

students not just for examinations, but to equip them with skills that transcend mathematics itself. Logical reasoning and problem solving are the most important skills, and mathematics can be the *shortcut* for a learner to achieve these.

However, at this point the situation became somewhat complex, as shown below:



The blue bar graph shows the cohort result in an exam-style assessment. It is a fairly "normal distribution", which we would expect from a "normal" cohort. The red bar graph shows the results from the Tessellation assessment. Almost a third of the students got results around 30%. In general these are students who were able to produce the mathematics, but were unable to communicate it clearly. So, we now know this is an area we need to focus upon.

Would we expect, no matter what kind of assessment we set, the results would be close to a normal distribution? This is what it should be if teachers seek to develop students' wider mathematical skills. Obviously this is not the case and many students tend to treat mathematics instrumentally, as a set of algorithms they need to follow to get from point A , the question, to point B, the answer.

As Skemp (1976) observed:

Pupils whose goal is to understand instrumentally, taught by a teacher who wants them to understand relationally… All they want is some kind of rule for getting the answer. As soon as this is reached, they latch on to it and ignore the rest. If the teacher asks a question that does not quite fit the rule, of course they will get it wrong.

Daniela conjectured:

a) Lesser-confident problem solving students are not ready to persevere once they become 'stuck'.
b) As with any other form of assessment, some students like it, while others do not.
c) Some learn from it more than others.

Overall, however, the teachers at SIS felt they learned more about their students than they might from a traditional type of assessment; some students experiencing the wonder of discovery:

"*It is interesting to note that this sequence of regular polygons with an increasing number of sides eventually would become a circle*." (Sophia),

Others students learned more about mathematical content knowledge:

"… other shapes as pentagon, heptagon, octagon,… cannot fit together perfectly as their internal angles are not factors of 360." (Cathal)

"I enjoyed this project because I could refer to my previous project to see where I went wrong, and I could recall my knowledge on factors (used in explanation) and I like using Geogebra." (Roxy)

Others learned something about themselves:

"*Besides learning a good deal about tessellations and why shapes tessellate, I gained some experience about how to organize myself and the consequences of not doing so."* (Xiao-ke)

"*Though I attempted to explain the formula for a regular tessellation I don't think I had enough detail implemented in the piece or fluency of language for people to make sense of it all.*" (Hannah)

It is worthwhile noting that some students gained a low score not necessarily because they would not be able to do better, but because they display a negative attitude towards this kind of assessment. Other students scored low marks because they could not coherently explain their thinking, whereas others did not see the mathematics behind patterns, for some students mathematics lies only in the textbooks. This 'new' mathematical experience appears to many students to be a different kind of learning; at issue is will we be able to make this a 'normal' mathematical learning experience? Only time and persistence will tell.

Ultimately Daniela wants SIS students to see mathematics as a creative discipline and this will depend upon what we demonstrate it to be and as Daniela says:
'*It is within ourselves, as teachers of mathematics, to make mathematics the queen of sciences and the art of patterns'.*

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References

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