A CLIL CONVERSATION ABOUT TEACHING AND LEARNING MATHEMATICS

Mike Ollerton and Patrick de Boer engage in a dialogue on the ‘how’ in mathematics teaching

Patrick: Hello Mike. Great that you want to help out and talk with me about mathematics and ways to implement it in the lesson. Before we start, could you tell us something about yourself and what you’ve been up to lately?

Mike: Hi Patrick and good to hear from you again. I am basically someone who is a passionate person about all kinds of things: football (Liverpool FC – I was fortunate enough to be at the 2005 European Cup Final in Istanbul against AC Milan), Bridge, hill walking, mathematics and my wife, though not necessarily in that order! I began teaching in 1971 and as a head of mathematics from 1986-1995 I guided a department to teach in active, problem-solving ways, and in all-ability groups, which was, and continues to be, most unusual in the UK. I have been self employed since 2006 and been involved in all types of projects, nationally, with schools inside and outside the UK and through courses and conferences, such as CLIL. My professional drive is to consider how to help teachers to cause their students to become active problem-solvers within the domain of school mathematics and to become less dependent upon textbooks.

Patrick: As an expert on teaching mathematics without textbooks, your methods could easily be implemented into CLIL lessons to make sure students talk and use English in classroom situations. What do you think about the statement that maths, because of its use of numbers instead of words, is one of the most difficult subjects to get the students to work on their English?

Mike: Because I believe in the value of learners writing about the mathematics they are currently working on I think mathematics is a perfect context to support students develop their English language skills. At issue is what different types of support they need regarding their mathematical vocabulary development. For example, as well as students learning to ‘add fractions’ such as $^1/_5 + ^2/_3$ I believe if students can explain *how* they add fractions then this supports their conceptual understanding. One aspect of this strategy is the use of key vocabulary such as: numerator, denominator, common denominator, equivalent fractions. Here the teacher needs to help learners identify such vocabulary and to use it in written sentences. A valid alternative is for a pair of students to collaborate to produce a *PowerPoint* detailing how to add fractions and subsequently give a short presentation to their peers. Again, here students are not only engaging with mathematics linguistically, they are also engaging with mathematics conceptually.

Patrick: Alright, so if I understand correctly you suggest students present their knowledge in front of the class for others, and themselves, to learn from. This is a great way to get them to use English, but how can you make sure the students also understand everything? To continue with your example, if they don’t know how to add fractions, will you help them in class?

Mike: The issue is that students only present what they know and are confident about. This could be a solution to a larger problem they have been working on, or it could be an explanation of a specific concept, such as Pythagoras’ theorem or how to add fractions. At issue is how students are taught in the first instance, so they understand the mathematics their teacher intended them to learn. Taking an example of learning how to add fractions, one way I have found valuable is to ask students, working in pairs, to each fold a piece of paper into five strips in one dimension and into three strips in the other dimension; thus ending up with the following:

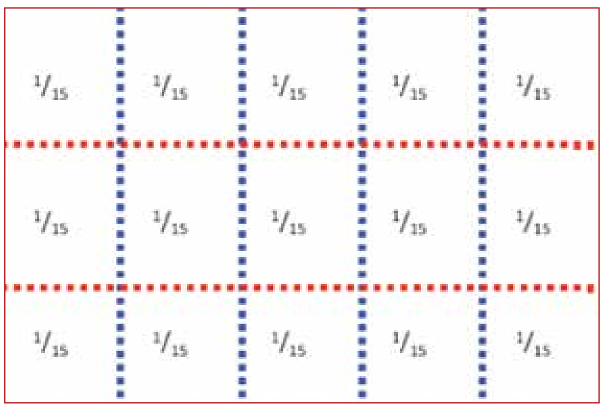


Figure 1: Students are then asked to write $^1/_{15}$ inside each piece, there will be 15 equal sized pieces.

Next ask one student in each pair to fold the paper so they can only see $\frac{1}{3}$ and ask the other student to fold their piece of paper so they can only see $\frac{1}{5}$. Then ask the pair of students to calculate $\frac{1}{3} + \frac{1}{5}$. Of course the answer will be $\frac{8}{15}$ and they will have '*arrived*' at this answer without me 'teaching' them how to add fractions! The next stage would be for students to make up, and calculate answers to, many problems involving adding so many thirds to so many fifths for example, $\frac{2}{3} + \frac{3}{5}$. Of course the same pieces of paper can be used for students to do some subtraction calculations.

Based upon my experiences, as a teacher, I have a strong belief that it is feasible to teach the whole of the mathematics curriculum in ways which enable learners to '*discover*' mathematical truths using a problem-solving pedagogy. At issue is the quality of the tasks the teacher offers in order that they can stop '*telling*' students how to perform certain skills, and instead provide them with powerful experiences through which learning becomes self-evident.

Patrick: Indeed, this way of teaching not only requires a great deal of experience, but also a great deal of preparation for a teacher if you want to do this every lesson. Besides, one of the reasons teachers might prefer a more traditional way of working is to make sure that some agreements are applied, such as the way to name a coordinate or the fact that a horizontal axis is named first. Do you think you can do without these '*rules*', having students *find out that themselves as well*?

Mike: There are three issues here. The first is that I would not expect myself to have to come up with ideas such as this every lesson. This is because I intend such ideas to have '*legs*'. By this I mean that over the course of the next lesson or two I would expect students to create their own addition and subtraction with fractions type questions in order to apply what they have learnt from the initial paper folding task and to consolidate their knowledge.

The second issue is about students explaining, in writing and with diagrams if appropriate, what they understand it means to add and subtract with fractions. In this way students are being asked to make their learning explicit, both to themselves and to their peers. By the way, using the same approach of paper folding, students can also learn how to multiply and divide fractions. This connects with the issue of the teacher providing students with key vocabulary as I suggested earlier.

The third issue regards students discovering '*rules*', or conventions. Of course it is important for them to understand there are certain conventions that are not '*up for grabs'*, such as the place value system or plotting co-ordinates on a graph. As such, and when appropriate, I am going to tell students what these conventions are… though having said that I might begin by asking students to see what happens if they intentionally plot some co-ordinates, the '*wrong way round'*, so instead of plotting (2, 3), they plot (3, 2). Helping students to recognise that this produces a mirror reflection in the line $y = x$ is one way to help students make sense of an important transformation. Likewise with the place value system, students need to know that the number 361 means 300 and 60 and 1, not 600 and 10 and 3 or 100 and 30 and 6. However, again here, there is a nice problem to work on, which is to find all possible three-digit numbers using the digits 3, 6 and 1 and to place these numbers in order from smallest to largest, then to explore the difference between adjacent pairs. So starting with 136, the next largest will be 163, then 316, 361, 613 and 631. By exploring the differences we gain 27, 153, 45, 252 and 18. An exploration of these differences reveals they are all multiples of 9… but why? Thus they are working with a given convention, but exploring a problem that will extend their understanding of the place value system.

Patrick: So exploring the use of conventions you talk about the values of numbers, cross over to probability assignments, and finish with a more advanced number issue. That's great! I can understand students being encouraged to learn maths if taught this way! You propose rather revolutionary ideas to change the way to teach students maths. What could a teacher who would want to apply this way of teaching start with?

Mike: Oh heck, that is both a difficult and easy question to answer! My difficulty relates to the different professional circumstances and contexts in which teachers exist. As a young teacher I was fortunate enough to have a first head of department who strongly encouraged his staff to use a range of investigative ways to teach mathematics; he helped us see how there were far better, more interesting ways of working with students than using textbooks. He was a strong member of the Association of Teachers of Mathematics (ATM) and he brought all the best ideas from the ATM into his and his colleagues' classrooms. So, for example from a publication titled *'Points of Departure book 3'* idea number 10 offers the following:

Skewed Pascal

Instead of starting Pascal's triangle with 1 1, followed by 1 2 1, what happens if we begin with 1 2 so the following arrangement is created?

1 2
1 3 2
1 4 5 2
1 5 9 7 2
1 6 14 16 9 2

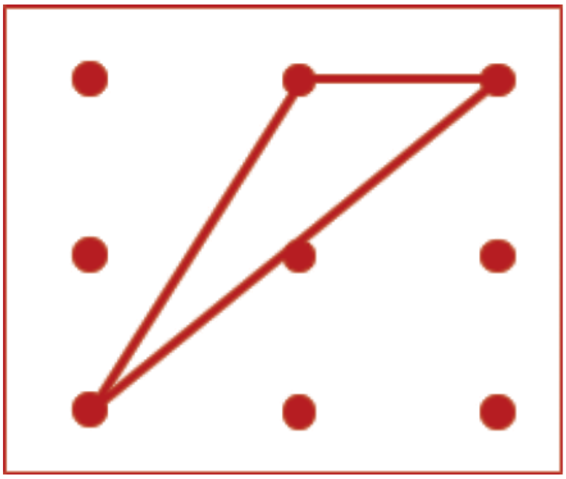
In what ever ways we usually explore number patterns in Pascal's array, looking perhaps at the linear and quadratic sequences, or summing values across the rows we can do exactly the same with skewed versions of Pascal. If we can start with 1 2 then what sequences are generated starting with 1 3, or 2 3 etc? In each skewed version we gain two different linear and two different quadratic (cubics, quartics etc) sequences and these are just ready for exploration and generalisation.

This is just one problem from a book containing 70+ other starting points and given there are four such resource books then there is clearly a wealth of ideas; I personally have used in the region of 100 of the ideas from these publications. These can be purchased as PDF downloads from www.atm.org.uk

When I became a HoD I also became a member of the ATM and, I guess, tried to emulate my mentor's philosophy. I used dozens more ideas from key resource publications in order to create a scheme of work which was entirely based upon problem solving approaches; eliminating the use of textbooks entirely. The easy part of the question to answer, therefore, is join the ATM, or the Dutch equivalent and search out the wealth of problems therein. There are of course very many other texts containing a wealth of puzzles and problems and I would happily provide anyone with such a list of authors/titles.

Patrick: The equivalent of the ATM would be "De Vereniging van Wiskunde Leraren" in the Netherlands. I really enjoyed our talk and think you're very inspiring to me and many other people. To finish up, could you speak about one activity every maths teacher should at least have done once, the one you are most enthusiastic about?

Mike: Thank you for these kind words; of greater importance, for me as a mathematics teacher though, is how through writing about one's beliefs in a conversation such as this helps to articulate one's pedagogy for teaching mathematics. Regarding my favourite activity for teaching mathematics, I would say this emerges from a particular resource called a *9-pin geoboard*, brought to the attention of the teachers in the UK by Caleb Gattegno, a famous mathematics educator, more than 60 years ago. The *9-pin geoboard*, together with an elastic band, is an excellent piece of equipment where learners can explore all kinds of geometric situations and the diagram below shows one triangle that can be made.



Students can explore how many '*different*' triangles can be made and having gathered a collection the teacher can pose a range of further problems such as:

- How do students know they have found all the different solutions?
- How can students prove this?
- What are the properties and the names of these triangles?
- What are their areas?
- What are their perimeters?
- Suppose we allow rotations or reflections (or congruent solutions) – how many triangles are there altogether?
- Suppose we explore quadrilaterals?

There are many other problems that can be devised using this simple piece of equipment, so for example if we take the shortest distance between any two pins as 1, then we can enable post-Pythagorean students to work with surds: $\sqrt{2}$, $\sqrt{5}$, and $\sqrt{8}$, or $2\sqrt{2}$ to describe perimeters of shapes. If we next consider a *16-dot grid*, then new surd values are possible, i.e. $\sqrt{10}$, $\sqrt{13}$ and $\sqrt{18}$, or $3\sqrt{2}$

Returning to the *9-dot geoboard*, we can think of it as a co-ordinate grid and pose a question about how many straight lines and what their equations are... A page of grids you might like to use for this and previous explorations can be found at www.atm.org.uk

Underpinning all such explorations is, for me, an important pedagogical consideration offered to the wider mathematics education community by Gattegno: *Subordinating teaching to learning*.

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http://www.clilmagazine.nl/

In the News

Sue Pope and Tony Cotton respond on behalf of the Association

I write as Honorary Secretary of The Association of Teachers of Mathematics (ATM) to register ATM's dismay at the programmes of study for mathematics published on 8 July. The ATM drew on the expertise of its members in order to compile a comprehensive response to the consultation and met with the Parliamentary Under Secretary of State Elizabeth Truss to discuss our response in detail. We feel that our thoughtful and evidence based response has been almost completely ignored.

We raised concerns about the overly ambitious yearly programmes which have many age-inappropriate expectations including premature formalisation. We also raised our concerns over the heavy reliance on practice as a principle teaching approach as this is will not lead to the development of fluent mathematicians. The curriculum as presented will result in more attention spent on developing technical competence in outdated written methods for arithmetic at the expense of developing secure foundations for progression through mathematical concepts and skills. Mathematical foundations which are compromised in the draft programmes of study include developing an excellent understanding of relations between number and quantities including place value; using mental methods as a first resort; the skills of estimation and equivalence; and most importantly the ability to reason mathematically and solve problems both as a means of learning new mathematics and developing understanding of mathematics through use and application.

The proposed changes will result in many children being labelled as failures in mathematics from an early age, as teachers attempt to cover the yearly teaching programmes. The presentation of the curriculum will not support primary to secondary transition as there are no clear lines of progression. Appendix 1, entitled formal written methods for multiplication and division, but including addition and subtraction as well as multiplication and division is a complete travesty and needs to be removed.

We would urge further revision to the programmes of study before taking them through legislation.

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