PLAYING AROUND WITH 30° 60° 90° TRIANGLES

Mike Ollerton invites Year 5 pupils to play with ideas.

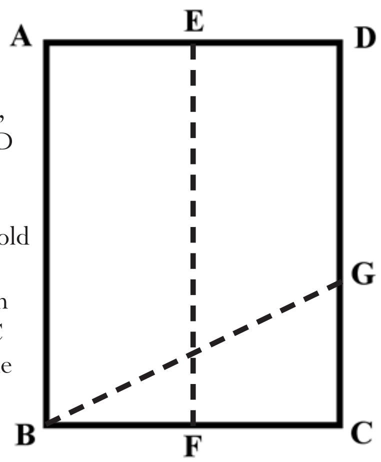
In recent years I have become interested in the idea of causing learners to 'play around' with ideas and resources. For example playing around with consecutive numbers using addition or multiplication or playing around with sequences: "If the first three values of some sequences are 2, 3, 5... how could different sequences develop?" This short piece of writing is about playing around with some 30° 60° 90° triangles. This emerged from an invitation to work in a primary school to provide some input into ideas for classrooms about 'shape and space.

Part of my brief was to spend the afternoon with a Y5 class where we did a good deal of shape-making through paper folding and considering properties and names of the shapes as well as creating tiling designs from A7 size paper.

The other part of my brief was to work with staff at an after school meeting to continue looking further at geometric ideas which teachers could adapt for use with their pupils. One idea I offered was based upon making shapes using 30° 60° 90° triangles. We began with 'the' standard way of creating such a triangle through paper folding. Now, I am fairly confident that if you are reading this article you will already know the two folds required to gain a 30° 60° 90° triangle... however, just in case, the two folds described below can be carried out on any rectangular size piece of paper.

The first fold, EF, bisects AD and BC

The second fold goes through corner B such that corner C folds onto line EF



Proving BFH and BCG are 30° 60° 90° triangles is probably beyond the scope of most primary school age pupils, however the business of playing around with several such triangles may not.

Once we have some 30° 60° 90° triangles we can pose accessible problems such as:

- How many different shapes can be formed by fitting any two of these triangle together by same edge length?
- What about three triangles?
- What are the properties, names and symmetries of these shapes?

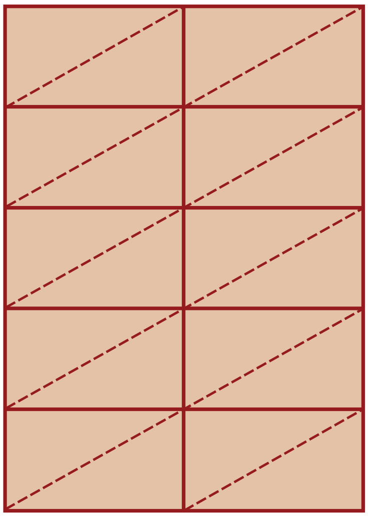
Some more challenging questions could be:

- If we label the three lengths a, b and c, what are the perimeters of the shapes?
- What shapes can be made if we abandon the rule of joining together by same edge length... and what conditions would we need to apply to avoid infinity?

It was this final problem which occupied a few minutes of my time last week; sitting at my desk I happened to have some 30° 60° 90° triangles cluttering up my workspace.

Possibly because I was in need of a few creative moments, I started to play to around with them. After a few minutes I came up with the following designs (overpage) and I wondered if anyone might be interested in seeing them and possibly coming up with a lesson, or two, based upon using 30° 60° 90° triangles.

Interestingly the triangles I used on this occasion started off as rectangles with sides in the ratio of $1{:}\sqrt{3}$, which I had only previously used for another paper-folding type purpose. I had generated these rectangles by dividing pieces of A4 paper into ten equal pieces as in the diagram below:



Mathematically speaking I should refer to these rectangles as being in the approximate ratio of $1{:}\sqrt{3}$. This is because dividing up an A4 sheet creates its own problem which is to see what the percentage error is of the smaller rectangles being in the ratio of $1{:}\sqrt{3}$ given the A4 sheet is in the ratio of $1{:}\sqrt{2}$

If you are prepared to accept the percentage difference is small enough not to worry about, in terms of creating a piece of practical equipment which is accurate enough to serve the purpose, then we can get twenty 30° 60° 90° triangles out of a sheet of A4 card without any waste.

The key issue, however, is less about playing around the 30° 60° 90° triangles and more about playing around with mathematics per se. Perhaps the phrase '*playing around with*' is synonymous with exploring ideas; about seeing what happens when, for example, learners play around with:

- square and triangular numbers;
- adding consecutive numbers;
- changing the variables *a*, *b* and *c* in the general quadratic $y = ax^2 + bx + c$;
- ATM regular polygon mats to create tessellations;
- Cuisenaire rods to make fractions;
- pegs and pegboards to make symmetrical shapes by varying the line(s) of symmetry;
- Diene's base 10 equipment to carry out computations;
- patterns on a 100 square.

Something I am interested in, and which frequently happens when I am fortunate enough to work alongside teachers in their classrooms, are the unexpected events which occur. These arise through observations made by class teachers relating to outcomes of unusual and surprising responses by certain pupils. This, in part, is because the teachers have opportunities to observe more and 'teach' less; it is also because the pupils are playing around, exploring ideas and trying things out... and possibly because there is never an objective written on the board.

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