

TEACHING AND LEARNING THROUGH PROBLEM SOLVING

Mike Ollerton relates some problem solving work with primary schools to DfES support.

The following piece of writing emerged from working in four primary schools in the West Midlands. The focus was teaching mathematics through problem solving. The schools had already begun to work through materials published on the DfES 'standards' site (www.standards.dfes.gov.uk), so I looked at these materials as part of my planning.

I noted the way problem solving was broken down into specific sections by DfES:

- finding all the possibilities
- logic problems
- finding rules and describing patterns
- diagram problems and visual puzzles
- word problems.

Each section contains suggested problems to engage pupils with each of these aspects of problem solving. Whilst I wholeheartedly celebrate the 'official' recognition of the role of problem solving to support the learning of mathematics, I am concerned about the prescribed way the materials are to be used, both in lessons and at staff meetings. However, as a way of bringing problem solving to the fore, this initiative is worth celebrating. In this article I hope to offer constructive ways forward and, at the same time, describe an interesting surprise event that occurred in a KS1 classroom.

Problem solving does embrace 'finding all the possibilities', 'logic', etc. I wonder, however, about breaking it down into specific steps or ways of working. Problem solving is rarely a clean, clear set of procedures, otherwise where would the 'problem' reside? Problem solving, by definition, is likely at times to be messy and ambiguous. When used effectively, problem solving becomes the vehicle for developing pupils' mathematics and

simultaneously helps develop autonomous decision-making and independence of thought and action. Problem solving also provides learners with a purpose or a context for learning mathematics. Finding all the possibilities and solving logic problems are only subsets of problem solving. I am far more interested in finding problem solving approaches that enable pupils to process, and therefore develop, their mathematical content knowledge.

For example, a problem such as 'Find all the whole number pairs that add together to make 10' can lead to pupils working on a variety of processes and concepts. Some of these are:

- ordering information or working systematically (processes)
- pattern spotting and generality (processes)
- determining that all possible answers have been found; and proof (processes)
- recognising commutativity, eg $6 + 4 = 4 + 6$ (concept)
- practising basic addition (concept)

If these pairs of values are then turned into co-ordinate pairs and then graphed, a range of other concepts can emerge, such as:

- working with the co-ordinate system
- drawing a graph
- the non-commutativity of the system of co-ordinates (except when $y = x$)
- negative numbers (if one of the pair of numbers is > 10)
- decimals (if non-integer values are allowed)
- finding the equation of the graph
- recognising that if the initial value of 10 is changed to 12 a parallel graph is produced.

The starting question can be made quite simple

and a variety of extension tasks are possible; thus, issues of access and depth have to be considered. These depend upon a teacher's working knowledge of their class.

I suggest there are five key criteria involved in setting up problem solving situations:

- 1 Any problem must be aimed at developing pupils' content knowledge
- 2 Problems need to be accessible, whilst at the same time puzzling
- 3 Use can be made of more 'open' type questions
- 4 To support differentiated learning, problems must be extendable
- 5 Independent learning is fostered.

1 Developing pupils' content knowledge

Mathematical processes help pupils make sense of mathematics. Problem solving simultaneously provides pupils with opportunities to develop knowledge and to practise and consolidate this knowledge. There are many examples where practice and consolidation occurs naturally whilst pupils are working on a problem; for instance, the 'Palindromes' problem, which works as follows:

Choose a 2-digit number (eg 57), reverse the number and add ($57 + 75 = 132$). If the answer is not a palindrome, repeat as before ($132 + 231 = 363$).

The answer is now palindromic and it took two addition calculations for 57 to generate a palindromic answer.

The problem is ... what happens with other 2-digit numbers?

Here pupils are practising addition whilst at the same time working systematically, classifying results, recognising patterns and seeking generality.

2 Finding accessible and puzzling problems

Solving puzzles and problems is a basic human characteristic. Using puzzles and problems in the classroom is an effective way of engaging pupils in learning. Access is the key to developing confidence, in order that pupils are prepared to tackle a problem without giving up before they have started. Finding starting points that all pupils can initially engage with enables them to overcome potential anxieties about not being able to understand something. Problems must contain an element of challenge and puzzlement, otherwise pupils are faced with triviality and a potential lack of stimulation.

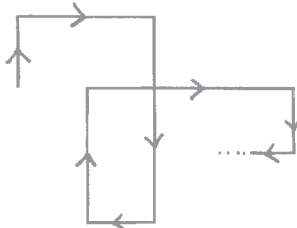
3 Using 'open' questions

I make a distinction here between open questions and open-ended situations. I define an open question as one that does not lead pupils to guessing a specific answer in the teacher's mind. eg 'What is 4×6 ?' This question can be opened up by asking, 'I have multiplied two numbers together and the answer is 24. What could the two numbers be?' I define an open-ended situation as a task which has several variables that pupils can choose to change. For example, the 'Worms' problem (ATM, 1983).

62. WORMS

Worms leave tracks in layers of mud.

A worm forms a piece of track, turns through 90° , forms another piece, turns, forms another piece, turns, ...



The drawing shows a 1, 2, 3 worm; it makes one unit of track, turns, makes two units of track, turns, makes three units of track, turns, makes one unit of track ...

Investigate worms.

In this problem, the variables are the number of values used, the relative sizes of these values and the angle that a worm turns through.

4 Extension tasks

Because pupils think and work at different speeds and to different depths, it is important to offer tasks that can be developed at different speeds and depths. Thus, an extension task to the question of finding pairs of numbers whose product is 24 could be to find pairs of values which are not integer values. A further development would be to write these pairs of values as co-ordinate pairs and observe the shape of the graph so formed. Devising and providing extension tasks is necessary to support differentiated

learning, which exists in all classrooms, irrespective of whether groups of pupils have been artificially constructed according to some tenuous notion of ability, or not.

5 Fostering independent learning

Problem solving is an important aspect of independent learning and helps create a shift away from didactic teaching. The more pupils are (through the tasks, puzzles or problems they engage in,) working evermore independently, the more effective learners they become. Through problem solving, we can create a culture of independence, where pupils are encouraged to make decisions and choices about how to set about a task. Consequently, the more independent pupils become, the more self-reliant and the more capable they become to make rational decisions about what they are learning and how and why they are learning something.

Speed, messiness and structure

Three further issues are those of speed, messiness and structure. All too often, mathematics is synonymous with pupils getting an answer as quickly as possible. For those who can speedily provide answers (usually in response to closed questions), mathematics may be an okay subject. However, for those who can't or choose not to enter into some kind of race to give an answer, the potential for them to turn away from mathematics should be a cause for concern. I believe there are good reasons for slowing pupils down, in positive, more reflective ways when they are engaging in mathematics; this also supports students to become more able to cope with ambiguity.

With regard to the messiness of mathematics, again pupils can be too often encouraged to produce neat and tidy work; recently I saw a pupil make one small mistake and because she did not want her page to be 'spoiled' she asked for another sheet to work on. In reality, problem solving or thinking something through are rarely neat and tidy processes; our thoughts 'jump about', sometimes making intuitive leaps and sometimes getting stuck on the 'easiest' of problems.

In a KS1 class, I offered a problem about making shapes using a $30^\circ 60^\circ 90^\circ$ triangle as the shape 'generator'. Pupils were each given two different coloured triangles made from card and asked to make other shapes by fitting the triangles together by matching edges. Having made some shapes, they were asked to count the number of sides of their new shape and to name them. Now, I

am certainly no expert at working with KS1 pupils; however, I reckoned that given an accessible problem and some discussion 'on the carpet' at the beginning, they would be able to engage with this task. The surprise arose as a result of the class teacher's observation of how one of the so-called 'lower achieving' pupils achieved more than her peers. We briefly discussed this and the class teacher suggested that because her pupils had been given a task where they could make mistakes and quickly move on to find 'correct' solutions, they were less conscious of needing to produce accurate results in the first instance. I have since contemplated this brief discussion and the implications of grouping such young children by notions of ability . . . but I am not going to get onto that particular soap-box just now.

The key issue for me is about setting up situations where all pupils can explore mathematics. Helping pupils to make sense of the structures within mathematics eg place value, the co-ordinate system, modulo arithmetic or angle/circle theorems, requires them to explore situations and discover mathematical truths. This is all part of sense-making and is a long way from producing answers to closed questions often contained in an exercise from a published scheme. Giving pupils lots of multiplication calculations to carry out may (and possibly may not) make them better at multiplication. However, when pupils understand both what multiplication means and when it is sensible to perform a multiplication, they become more effective mathematicians and not just more effective multipliers.

Mike Ollerton is a freelance consultant (mikeollerton@btinternet.com).



Reference

ATM (1983) *Points of Departure 1*, Idea 62, ref ACT001

DICK TAHTA

The obituary to Dick will appear in the May issue of *Mathematics Teaching*.

We invite educators whose work was influenced by him to write for future issues.

The editors email addresses are on page 1.

The attached document has been downloaded or otherwise acquired from the website of the Association of Teachers of Mathematics (ATM) at www.atm.org.uk

Legitimate uses of this document include printing of one copy for personal use, reasonable duplication for academic and educational purposes. It may not be used for any other purpose in any way that may be deleterious to the work, aims, principles or ends of ATM.

Neither the original electronic or digital version nor this paper version, no matter by whom or in what form it is reproduced, may be re-published, transmitted electronically or digitally, projected or otherwise used outside the above standard copyright permissions. The electronic or digital version may not be uploaded to a website or other server. In addition to the evident watermark the files are digitally watermarked such that they can be found on the Internet wherever they may be posted.

Any copies of this document MUST be accompanied by a copy of this page in its entirety.

If you want to reproduce this document beyond the restricted permissions here, then application MUST be made for EXPRESS permission to copyright@atm.org.uk

*This is the usual
copyright stuff -
but it's as well to
check it out...*



The work that went into the research, production and preparation of this document has to be supported somehow.

ATM receives its financing from only two principle sources: membership subscriptions and sales of books, software and other resources.

Membership of the ATM will help you through

*Now, this bit is
important - you
must read this*

- Six issues per year of a professional journal, which focus on the learning and teaching of maths. Ideas for the classroom, personal experiences and shared thoughts about developing learners' understanding.
- Professional development courses tailored to your needs. Agree the content with us and we do the rest.
- Easter conference, which brings together teachers interested in learning and teaching mathematics, with excellent speakers and workshops and seminars led by experienced facilitators.
- Regular e-newsletters keeping you up to date with developments in the learning and teaching of mathematics.
- Generous discounts on a wide range of publications and software.
- A network of mathematics educators around the United Kingdom to share good practice or ask advice.
- Active campaigning. The ATM campaigns at all levels towards: encouraging increased understanding and enjoyment of mathematics; encouraging increased understanding of how people learn mathematics; encouraging the sharing and evaluation of teaching and learning strategies and practices; promoting the exploration of new ideas and possibilities and initiating and contributing to discussion of and developments in mathematics education at all levels.
- Representation on national bodies helping to formulate policy in mathematics education.
- Software demonstrations by arrangement.

Personal members get the following additional benefits:

- Access to a members only part of the popular ATM website giving you access to sample materials and up to date information.
- Advice on resources, curriculum development and current research relating to mathematics education.
- Optional membership of a working group being inspired by working with other colleagues on a specific project.
- Special rates at the annual conference
- Information about current legislation relating to your job.
- Tax deductible personal subscription, making it even better value

Additional benefits

The ATM is constantly looking to improve the benefits for members. Please visit www.atm.org.uk regularly for new details.

LINK: www.atm.org.uk/join/index.html