SERIOUS LEARNING, FUNNY MATHEMATICS AND MATHEMATICAL FUN

Mike Ollerton

Learning mathematics is a serious business. For some children who perceive mathematics as a set of difficult, tedious and pointless exercises and, as a consequence, are unable to use mathematics at a basic level in their adult lives, it is a serious business. An important aim, therefore, in teaching mathematics is to make it accessible. Access is achieved in different ways for different people; some ways of becoming motivated to learn mathematics arise through interest, challenge and pleasure. This article explores ways of helping to make the learning of mathematics accessible through problem-solving and through enjoyment.

Funny mathematics

When my children were in their teens they described the way they understood I taught mathematics as '*Funny mathematics*'. I took this as a compliment: a sign of affection. It was also their way of characterising the differences in their perception of how I taught mathematics, using methods such as 'people maths', different types of equipment and without the use of textbooks, by comparison with the way they learnt their mathematics which largely involved textbooks and worksheet- or scheme-based approaches. Watching them develop a feigned indifference, at best, for mathematics, I became aware of my own motivations for making mathematics a more enjoyable set of experience for those I taught. I believe, however, they yearned for their teachers to teach them funny mathematics.

Mathematical fun

There is a 'game' I regularly play with new groups of students of a wide range of ages. The game involves everyone sitting in a ring and each person being given a different number from 1 upwards. I stand in the middle, or in a gap at the edge of the ring and call out pairs of numbers, whereupon these people are asked to change places. This develops into calling out sequences or sets of numbers according to certain rules and everybody who holds a number in the sequence or the set has to swap places with another person. Some sequences or sets of numbers might be: even, odd, prime, square, less than 5, between 14 and 22, factors of 12, Fibonacci, one more than a number in the 3× table, etc. After a few movements I sit on one of the chairs, thus leaving another person without a chair. This person then has to make up a movement and the game continues.

What always amazes me about this game is that certain humorous events always happen. The first event is that when I sit in one of the chairs, thus creating a situation where someone else is left without a chair, there occurs a slow, dawning realisation that they are now the person in the middle and must, therefore, make up and call out a rule in order to create the possibility of gaining an occupied chair. The second event is the enormous amount of fun and laughter that ensues, whether the participants are young children or mature adults. Despite the fact that somebody ends up in the middle without a chair or only as long as it takes them to call out another movement rule, there becomes a growing sense of purpose of wanting to get a chair and not be left in the middle.

Identifying and accessing mathematical concepts

The last time I played this game was with a delegation of elementary and high school principals, a special needs county co-ordinator and a professor of education from Virginia, USA. This activity was one of several aimed at developing problem-solving approaches to learning and teaching mathematics.

Other tasks we worked on were:

(i) What happens when an isosceles right-angled triangle (Irat) is successively folded, once, twice, three times etc., along the line of symmetry then cut down the final line of symmetry.

(ii) Everyone stands in a ring and each person has a number from 1 upwards. Number 1 sits down, then every *other* person sits down; this continues several times around the ring until there is just one person left standing. At the outset this task is described as trying to determine who will be buying the beer!

(iii) Also in a ring, a ball of string is passed around, using different pass rules; the first person holds on to the loose end and the second person wraps the string around a finger and passes on using the same rule. Different shapes are formed according to the number of people in the circle and the size of the pass rule. So, for example, with twelve people and a pass rule of 5, a twelve-pointed star is formed; with a pass rule of four an equilateral triangle is formed etc.

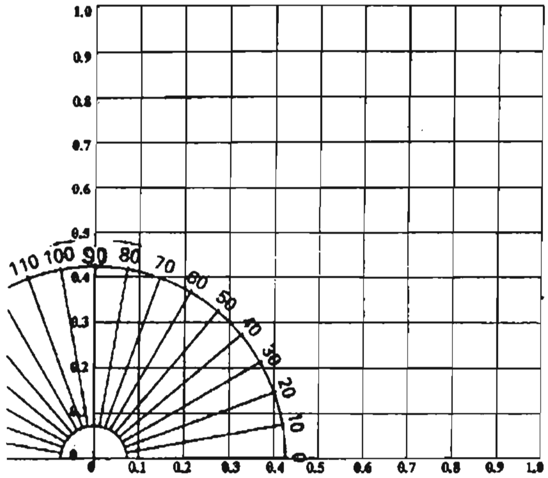
Each task held a certain amount of amusement and surprise, but central to all tasks was the potential for engaging with different mathematical concepts and practising existing skills. In the moving places game concepts of sequences, and the language of mathematics emerges. The 'Irat' task provides scope for working with the shapes that are formed, conservation of area, adding fractions with denominators of 2n and the number of pieces form. The *Who buys the beer?* task is a good context for developing knowledge of the significance of binary values. The *Pass the ball of string* game is useful for naming shapes, working with divisors, and calculating angles of the shapes on the circumference of a circle. I have not listed all the possible concepts and skills here; the key issue is that it is both feasible and desirable to construct humorous and surprising learning situations, so that learners can gain pleasure from learning mathematics. I was particularly aware of one delegate's comment after the session which was that he had worked on more mathematics in the previous hour than he had done for many years. The poignancy of this comment was that as a child he had grown up with expectations that he should be interested in and good at mathematics. This, however, had not come to fruition.

The last but one time I played this game was with more than 30 Y7 students at a school in the Midlands. The context was an in-service training day with teachers in a mathematics department. I had previously agreed with the head of department that, during the day, I would spend three lessons working on ideas with staff and three lessons working with timetabled mathematics groups: these were Y10, Y7 and Y8. Each lesson provided opportunities to put key pedagogic principles into practice, these being inclusion, active learning and equipment-based approaches to learning. Of course, each principle was not exclusive to a particular lesson, but they were significant pedagogic factors implicit within each lesson.

Inclusion and starting points

In earlier planning with the head of department I agreed to work on an introduction to trigonometry with Y10 students. At first it was suggested I work with the 'top' set. However, in order to develop the principle of inclusion, that everyone in Y10 should be provided with an entitlement to access trigonometry, irrespective of their so-called 'ability', I asked about the possibility of there being a room large enough to accommodate both 'top' and the 'bottom' set groups. This was possible and so I planned a trigonometry starter lesson for approximately 50 students.

This starting point utilises an overhead projector, showing a rotating arm of length 1 with the centre of rotation at the origin of a co-ordinate grid; a protractor is overlaid.



In order to create mixed groups I identified nine work areas in the room and as the students filed in I gave each a number between 1 and 9 on a piece of card and asked them to sit at the desks with the same number. Once they were all assembled I explained we would be working together and their small-group responsibility was to ensure that everyone in the group ended up with the same measurements. At this stage, I made no mention of the word 'trigonometry' but instead emphasised the ground rules of co-operation. I then asked the students to think about three 'basic' mathematical ideas which they would all be familiar with, these were:

- angle is a measure of turn,
- a line starting at 0 and finishing at 1 can be split up into ten decimal divisions, 0, 0.1, 0.2... 0.9, 1, and
- on a coordinate grid the convention for moving is horizontal first and vertical second.

Of course for some students these would be well-understood ideas, for others this context would provide them with a further opportunity to work on them in a combined way.

The task was to record the coordinates at the end point of the rotating arm for every 10° measure of turn, as it rotates from 0° on the horizontal to 90° at the vertical. Because the scale of the axes is measured to an accuracy of 1 decimal place, the resulting answers can be written to 2-decimal places.

Demonstrating the starting point took only a few minutes, and quite soon each of the nine groups were collecting the information. By the end of the lesson some students were able to identify connections in their results, such as the condition under which horizontal readings were the same as vertical readings. Others had identified the angle when the horizontal and vertical reading were the same. As this was just a starting lesson it was important to lay out a possible framework which students could use to develop their understanding of trigonometry. This would begin to emerge when, at a future time, students are asked to use a calculator to establish the cosine and sine of those angles for which they had already worked out the coordinates, then compare the two sets of results.

The central pedagogical principle is that everyone, whether in the 'top' or 'bottom' set, can be provided with access to a concept which, traditionally, is only taught to the most 'able' students, however they are defined.

Active approaches to learning mathematics

Trying to find ways of getting students to leave the comparative safety of their desks, in order to work on mathematics, requires active involvement. One way to achieve this is to use '*people maths*' approaches. The well-used ***Frogs*** task is one example of people maths. The circle game, as described above, is another. Trying to ascertain the essential elements in active learning that make it beneficial to understanding mathematics is worthy of analysis. Of course, some students will not like being active and one could argue that such approaches are unattractive to such students. However, the same argument could be applied to any form of learning, such as investigative, rote, worksheet, page 27 exercise A, jumping up and down 20 times as fast as you can . . . The key issue is to try to provide a wide range of teaching methods to match different learning styles. The *Who buys the beer?* or perhaps for younger students: *Who wins the prize?* problem, is another example of a *people maths* problem.

There are a number of *people maths* problems and, with a little ingenuity, it is possible to play out many traditional mathematical ideas. For example, asking students to stand on a number line and, whether the aim of the lesson is to work on decimals, directed numbers or rational and irrational numbers, a *people maths* approach might be used. Lots of opportunities exist for demonstrating loci ideas using *people maths*, and it is quite feasible to ask some students to 'walk' out the path of the 'snooker' ball, when setting up the well-known ***Snook*** problem.

Active approaches are useful for: whole group involvement (or several smaller groups carrying out a similar task), allowing mathematics to exist and be constructed beyond the pages of a textbook, discussion between students, and encouraging individuals to see, feel and be part of a mathematical scenario.

Equipment-based approaches to learning mathematics

The most obvious benefit that using equipment has for learning mathematics is that ideas can be constructed from concrete beginnings and later translated into abstract understandings. I classify equipment as anything which the student can hold, fold, manipulate, cut, re-shape and re-organise in some way. As such, a piece of paper, which can be used to make different shapes, is as much a piece of equipment as a geobard, a set of Cuisenaire rods, a protractor, multi-link cubes, etc. Of course, some equipment might be used on a more regular basis, such as a ruler or a protractor. What gives value to any piece of equipment is the way it is used and the purpose for which it is being used.

In the example above when the isosceles right-angled triangle is cut down its line of symmetry there are opportunities for a range of possibilities. These are: the language of mathematics in terms of similarity, congruence, fractional areas and transforming the two half size triangles into a square or a parallelogram. After making one fold and then cutting down the line of symmetry, the shapes created can be discussed: a square with an area of a half of the original piece and two smaller isosceles right-angled triangle with areas each of a quarter.

The number of shapes formed after successive folds followed by one cut are:

Folds	0	1	2	3	4
Pieces	2	3	4	6	9

There are obviously rich opportunities for trying to see how the number sequence develops.

By providing students with the equipment, they can get 'closer' to what is happening and, therefore, closer to the mathematics. Of course, having a hands-on experience cannot guarantee that students will understand all of what is happening. However, without such experiences and if the ideas stay only as words on the pages of a textbook or on a worksheet, then students are one stage removed from the action.

In these days, however, everything needs to be measured, so that targets can be set, league tables drawn up, learning outcomes inspected and value for money ascertained. It is, therefore, increasingly difficult to see how anything which cannot be directly tested and measured, such as students' attitudes to learning and students' enjoyment of learning, can be given priority. How can passing tests and mathematical fun co-exist? What is certain is that students will be given tests to measure their so-called learning; market forces and political dogma demand this. What is up for grabs is whether this learning can take place whilst students are simultaneously having some fun.

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