Mike Ollerton has been reading the LMS report and finds some support for his goals for school mathematics.

PROOF, PROGRESSIVENESS AND PROCRASTINATION

The other day I was discussing, via Inter-Planetary Mail, issues of being a mathematics teacher with Martia, a Martian friend. She was failing to understand how anyone in their right or left minds could volunteer to take on this role in society.

Martia: "Isn't mathematics the subject that lots of people say they can't do, could never do, fear, dislike and sometimes hate? And aren't mathematics teachers constantly being told that they are getting it wrong by adults and by the media, and being asked by their pupils:'Why are we learning all this stuff?' "

Mike: "Well ... yes, but teaching can be enjoyable and fulfilling and teaching mathematics through problem solving can lead to really interesting and thought-provoking ideas."

I told her about a problem I had worked on with a PGCE group the other day based upon consecutive sums and how some students had come up with a new (to me) theory that any power of two cannot be expressed as the difference of two non-consecutive triangular numbers and how exciting I had found this.

Martia: "This all sounds very well but how do teachers cope with the constant criticisms of falling standards and of children not being as good at mathematics as they used to be?"

Mike: "Well, children's achievements are based upon how many As, Bs and Cs they get or fail to get after eleven years of compulsory schooling and this depends upon where examiners draw the grade boundaries."

Martia: "You mean to say that each child leaves school with a number of grades next to their name ... but what about those children who don't get very high grades?"

Mike: "Well, we tell them that the examination system only allows for a certain amount of higher grades, so some people have to get lower ones."

Martia: "Do you mean your system, in creating successes and winners, by the same token creates failures and losers?"

Mike: "Well, we don't tell them that of course ... we might tell them to jolly well pull their socks up!"

Martia: "So why is the Government unhappy that only fifty percent of children reach the average standard?"

Mike: "Well, because ..."

Martia: "Surely wherever an average line is drawn this automatically means fifty percent of what or whom is being measured would be below the line and fifty percent would be above."

Mike: "Well, we must find fair and equal ways of measuring children and their achievements."

Martia: "So why do anonymous bureaucrats take all the responsibility for deciding children's achievements instead of teachers and children themselves?"

I began to get an inkling of what lay behind her questions and her inability to make sense of what happens in my own small corner of planet Earth, when she told me that on Mars the most important qualities are honesty, integrity, caring for others and personal responsibility and these qualities are valued and integrated into every aspect of the Martian curriculum.

Martia: "I suggest you write down your understanding of the situation so that we might both have an opportunity to unravel and understand some of the issues teachers of mathematics are faced with."

As the debate over standards of teaching and learning mathematics necessarily continues and hopefully develops beyond the rhetoric and nostalgia for a bygone, golden age that somehow passed me by, I find myself increasingly frustrated about:

- the advice teachers are offered from many sources;
- the way 'progressiveness' has become misconstrued as malpractice;
- the mechanisms used to 'measure' and quantify children's mathematical achievements.

Proof

Debate over standards has always been with us. Each generation is naturally concerned about the next and whether the standards by which we were measured are being maintained. The London Mathematical Society (LMS) report [1] is part of this ongoing debate. The report is critical of certain aspects of school mathematics, particularly with regard to students who 'need to continue their mathematical training beyond school level'(p2) suggesting that 'many 'high-attaining' students are seriously lacking in fundamental notions of the subject.' (p5).

There is, however, a recognition that such problems affect all children. 'We need to improve the mathematical foundations laid at school level for all our students, whether or not they are likely to proceed to higher education.' (p5)

The report's main concerns appear in paragraph 5 of the summary (p2) and relate to: the perceived decline in fluency of numerical and algebraic accuracy; the marked decline of students' analytical powers; and the shift away from a perception of the place of precision and proof in mathematics. It would perhaps have been useful if notions of exactly what 'proof' means to the authors of the LMS report had been provided. Proof occurs at different levels of sophistication according to the task being undertaken and the age and maturity of a person.

Proof to a toddler means that if falling over hurts, then you try not to do it quite so often. A proof from a twelve-year-old that all the possible triangles on a 9 pin geo-board have been found will undoubtedly be different to the proof a seventeen-year-old following an A-level course might write. Proof, just like any other type of process skill, is used and developed in many guises and to various levels of complexity. However, if in general students cannot analyse information, be fluent with algebra or be able to construct a proof relevant to a situation, and if in the past many more students were able to acquire and demonstrate such skills, then clearly 'something must be done'.

In order to make realistic comparisons between generations of learners of mathematics and try to prove that today's learners know less about proof than those of yester-year, it is important to look, not just at those people who were perceived as the successes of a system of education, but also at the majority of people for whom the same system of education was an abject failure.

Progressiveness

If ever a word raised the emotional angst and caused polarisation! The politicisation of progressiveness is based upon subjective notions that new methods can never match those from the past. All too often the battleground is drawn between the progressives ('letting it all hang out man') and the traditionalists ('back to basics') and polarity occurs. I am interested in finding ways of bringing these poles together, so that the best methods progress offers can aid the learning of traditional concepts such as Pythagoras' theorem and trigonometry. How anyone can be upset by the notion of people making progress is beyond me; we are surely not interested in 'regressiveness'. My definition of progressiveness is the advancement of effective ways of teaching and learning mathematics within social classroom contexts.

If teachers' autonomy and responsibility for making decisions about how to become more effective are constantly undermined by criticism from academics, journalists and politicians; if teachers are encouraged to adopt published schemes of work, often written by ex-practitioners, claiming to 'cover' the national curriculum and are then subjected to systems of examinations driven by market forces, is it surprising that children's learning of mathematics is seen to be at a low ebb? Paul Ernest [2], gets to the heart of issues of underachievement, by attacking three, so-called 'progressive' approaches to teaching. These are: the growth of individualised learning schemes; the adoption of the 'spiral curriculum'; and the methodology of teaching mathematics through unrealistic applications.

The growth of individualised schemes

I share Ernest's concerns about individualised schemes. Pupils, by following an individualised scheme are directed through narrow, uninspiring pathways, thereby thwarting the very individuality such schemes intend to support. 'In theory, each child can study an individual programme tailored to meet his or her particular needs. However, in practice, teachers do not have time to identify each child's needs. Furthermore the so-called individualised schemes usually only offer two or three routes and so they rarely meet children's individual needs. Learners just follow one of these two or three lonely furrows, without the inspiration of a teacher's teaching. They also miss the stimulation of sharing their maths with others working on the same problem.'

The dangers are that teachers become managers of the scheme and by doing so their responsibility shifts from meeting pupils' needs to administering the scheme. 'I think that giving the so-called individualised maths schemes responsibility for teaching, instead of giving it to the teachers themselves, is profoundly demotivating and damaging for learners.'

I believe individualised learning schemes are a failed attempt to replace many text books that previously offered learners little more than a control

mechanism for carrying out repetitive tasks. By adherence to a scheme teachers are discouraged from engaging in curriculum development

Adoption of the 'spiral curriculum'

Ernest's description of the spiral curriculum is one that exists in many classrooms today. 'The spiral approach breaks the mathematics curriculum into several dozen tiny topics, such as Decimal Operations, Ratio, Percentages, Brackets, Reflections Vectors and so on... The idea of a spiral curriculum is that by returning to the topic again and again... at progressively more difficult levels, integration and continuity in learning is achieved.'

This approach leads to fragmentation of the curriculum, where separate skills are taught and practised in isolation and rarely connected together. Versions I and II of the National Curriculum encouraged fragmentation by the way it was laid out in hundreds of separate statements of attainment. A further force for fragmentation is the form of assessment and the style of questions used in GCSE examinations. I shall return to this later.

'Child-centred' approaches are condemned as the root cause of the destabilisation of our education system. Again, teachers are faced with polarity. If child-centred approaches are only about children 'discovering' concepts for themselves, then I would also be dubious of this approach. My understanding of a child-centred approach is a mixture of children exploring ideas and solving problems, with the teacher deciding sometimes to intervene by telling a child something and at other times making a professional judgement to stand back. The idea that a child-centred practitioner never 'tells' a child anything is nonsensical; on the other hand, if all the teacher did was to tell children how to do everything or offer teaching that was only about drill, rote learning and 'whole class' teaching followed by the practising of the skills from exercises, this would be equally fruitless. A recent experience confirmed for me the futility of such an approach. I gave a 'top' set Y9 class a problem about constructing nets for square-based pyramids. The solution to this problem would have been aided by their drawing upon and recalling Pythagoras' theorem, which the class teacher said they had 'learnt about' the previous week. Learning Pythagoras, in this case, meant that the pupils had done lots of calculations in their exercise books about working out lengths of sides of right-angled triangles taken from their text books. Whilst the pupils set about the pyramid-making task purposefully, only two or three were able to recognise a need to use Pythagoras to work out the necessary measurements. A further dozen children were able to make progress when the class teacher and I suggested they think about the Pythagoras work they had previously done. The rest of the class weren't able to recall and apply Pythagoras to the problem, even when they were explicitly advised to do so. I am sure that many teachers will recognise this kind of scenario, and how this leads to much wringing of hands. But what is the solution? Does it mean that the pupils need to do more exercises? Does the teacher need to spend more time telling them again ... in a louder voice? Should the teacher require the pupils to chant: "*The square on the hypotenuse is equal to the sum ...*" Excuse me I'm just receiving an IP-mail.

Martia: "Michael, stop being ridiculous"

Mike: "Sorry, but sometimes it seems this is what some people expect us to do."

A further aspect inherent in the spiral curriculum is the belief that mathematics is a hierarchial discipline, and certain harder concepts cannot be tackled until easier building blocks are in place. I challenge this idea, and surmise the authors of the LMS report do from the following quote, which I fully support: 'Is it not true that the easier subjects should precede the harder? On the contrary, some of the hardest must come first because nature so dictates and because they are essential to life. The first intellectual task that confronts an infant is the acquirement of spoken language. What an appalling task: the correlation of meanings with sounds! ... All that I ask is that, with this example staring us in the face we should cease talking nonsense about postponing harder subjects.'

Ernest's third line of attack upon 'progressive' methods centres upon the way mathematics is taught through applications. 'The applications of maths used in texts are notorious for being unrealistic ... the biggest problem with applications in school maths is that they conceal the mathematical concepts and skills they are meant to be teaching. If most of a pupil's work on a topic, ratio say, is disguised in terms of 'realistic' problems, the central concepts and methods will also be hidden, and may never be identified.'

As a GCSE scrutineer for SEAC some years ago, I became painfully aware of the kinds of phoney contexts that examiners wrote into their questions and used in order to (fail to) attempt to test children's mathematical knowledge. A typical example was a question about vectors based on boats moving on a square grid drawn between the Scottish mainland and one of the Western Islands!

Both Ernest and the LMS report criticise the use of poorly focused investigations and problem-solving tasks. Again 'investigations' have grown in misconstrued ways. The institutionalisation of investigations, arising primarily from the famous 243 paragraph in Cockcroft and later as Ma1 in National Curriculum, has led to their use in mathematics classrooms mainly as assessment tools and more often than not as bolt-on parts, taught separately from the rest of the mathematics curriculum. Instead of 'doing' investigations, in-

vestigative methods need to be integrated into children's learning of mathematics. This means using the best methods progressiveness offers to support children's learning of traditional mathematical concepts. Ernest says 'the very best progressive maths teaching involves problem solving and open ended investigatory work. When this is done by enthusiastic and skilled teachers in supportive school environments, the outcome is to develop real mathematical thinking skills and competencies in pupils. This is maths teaching at its very best. Unfortunately good progressive practice of this sort is all too rare in the maths classroom. Plenty of problems and investigations are done, but often not in the way that leads to real understanding and the growth of mathematical knowledge. So the problem with the best progressive practices is that they are all too rarely done properly.'

In a similar vein the LMS report suggests that 'progress in mastering mathematics depends upon reducing familiar laborious processes to automatic mental routines, which no longer require conscious thought'.

How can this be achieved so that children become interested and motivated to learn mathematics instead of submitting to mechanistic ways of working that stifle real understanding? How can children develop mathematical thinking skills so that they recognise when it might be appropriate and useful to draw upon aspects of their knowledge to solve problems? How can practising routines be justified if at the very point of a need to use a routine, the learner is seemingly clueless?

The challenge is adequately framed in a question I have heard Kath Cross (HMI) ask at conferences; "*Where are the BIG ideas in mathematics education?*". I believe one response to this challenge is to construct coherent modules that draw upon investigative problem-solving approaches and simultaneously practise and consolidate several content skills. At various points in a module children need to be made explicitly aware of the specific content skills they are using and developing. This can be achieved through problems that pupils are frequently asked to engage with. For example, exploring the different areas that rectangles with a constant perimeter can cover potentially engages the problem-solver with a range of concepts, such as perimeter, dimensions, area, devising formulae, drawing graphs and maximisation; together with subordinate skills such as using whole numbers, decimals and symbols. Different perimeters can be explored as can different shapes, thereby opening up opportunities for curriculum differentiation.

Procrastination

In MT153 (***An unexpected dream***) I criticised the current state of the examination system. I continue to do so. Examinations, through the types of questions, fragment and undermine the curriculum; this inevitably has a demeaning effect upon the way mathematics is taught and learnt. The misconception some politicians have is the existence of a correlation between the number of questions a child answers correctly on a test and what that child has learnt *and understood*. This puts far more importance on the value of tests than they deserve. The testing edifice, put in place to justify and uphold a free-market ideology in education is disingenuous and will serve only to undermine future generations of children's educational aspirations and opportunities. Indeed, such is the pressure on schools to compete in the free-market via league tables, that a specific section about market forces appears in the LMS report: 'Some of the strongest criticisms brought to the attention of the working group hinged upon consequences of the simplistic application of 'market forces'... the use of unnecessarily crude measures of success often militates against what is ostensibly desired.'

The style of teaching required to prepare children for tests is the very style that leaves children knowing little about mathematics beyond practising those routines required in order to answer the kind of questions they are likely to face in the test.

Martia: "Aren't you being rather negative in your appraisal of the state of teaching, learning and assessment of Earthling mathematics? I thought you were really enthusiastic about mathematics ..."

Mike: "I seek not to attack teachers, but to identify the leashes by which testing constrain and direct us."

Teaching to the test or to the style of assessment is firmly entrenched. I recall a comment from a meeting some years ago in Telford when one teacher stated that prior to giving his class the ***Octagon loops*** KS3 assessment task, he would first of all teach them about sequences, number patterns and functions, so that when they needed to spot 'the' pattern arising from the problem pupils would be more likely to come up with the required generalisation ...

A justification for testing is that it (supposedly) provides a common measure of children's learning and therefore of teachers' teaching. Consequently, teacher assessment, in a free-market oriented system, cannot be trusted. This regrettably, has led to the reduction of the weighting of assessment by coursework to a mere twenty per cent maximum. 'Assessment remains the major difference between vocational awarding and traditional examining. One hundred and fifty years of examining in Britain have

taught us to love and trust an 'end-loaded' system. Put simply, all the assessment is put at the end of the course by outsiders who are apparently more to be trusted than teachers or lecturers. The downgrading of coursework in GCSE in 1992, after it seemed almost too successful, is a good example of this belief – a belief not shared by most of our European Union partners. If, on the other hand we are to trust the teachers' assessment, then stringent pre-conditions have to be met before this can be accepted. This is sometimes known as a 'front-loaded' model. Assessment drives teaching and learning methods and few teachers doubt that the use of projects, coursework and oral and practical components, give much greater depth and breadth to the learning process.' [3]

The LMS report notes that 'the impact of mere curriculum challenge ... will be limited if other aspects of the educational system are not also subject to serious scrutiny' (p5) and takes this further by arguing 'we would all like to see standards raised, to drive out bad practices and to foster good ones. However, the use of unnecessarily crude measures of success often militates against what is ostensibly desired.'

My interpretation of 'serious change', 'crude measures of success' and 'bad practices', relate to the way children's mathematical knowledge and understanding is measured and how their achievements fail to be celebrated. If we are serious about changing the education system in order to make progress, this must mean scrutinising the examination tail that wags the curriculum dog.

"Dear Mike ... curriculum dog? Martia × ×"

References

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