Mike Ollerton wonders if there is any room left for surprises in a National Curriculum dominated by examinations.

AN UNEXPECTED DREAM

At the 1994 Edinburgh festival I was privileged to see a production of A midsummer nights dream by a Russian theatre group – Podol Art Project.

Because I 'know' the play – knowing for me in this instance means having seen ten or so productions and played the part of Wall in a school play – it did not really matter that this production was spoken mainly in Russian and that there were few props. The set was a bare stage and the costumes consisted of long flowing silver-grey cloaks, with the mechanicals wearing scarlet.

In all, the play could be described as minimalist. My reasons for feeling a sense of privilege at seeing this production were:

- the actors were full of energy;
- they appeared to be fully enjoying the parts they played;
- I was enabled, if only fleetingly, to cross cultural and language barriers;
- they brought a new and unexpected interpretation to the play;
- my applause at the end was spontaneous.

It is fairly common practice for Hippolyta and Titania, the underground queen, to be played by the same actor and the Podol production used this format. However, one particular event stuck out for me and took place during the mechanicals' rendition of the 'Tragedy of Pyramus and Thisbe'. During this scene there was a beautiful sequence of events, the like of which I had never seen before.

Hippolyta, when she was Titania the underworld queen, had fallen in love with Bottom in his transformed state as an ass. At the wedding feast at the palace of Theseus, as the mechanicals play out their 'merry and tragical, tedious and brief' play, Hippolyta recognises Pyramus as someone she has met before somewhere. She is drawn towards him, yet seemingly cannot understand what emotions are arousing her interest in this foolish actor. Bottom also 'vaguely' recognised Hippolyta and as the 'tragedy' unfolded, there grew a powerful yet unrequited love scene between them. This was transmitted by glances and stronger recognitions that in some other world they had met as 'lovers'.

I remember being astonished by the way that new levels of understanding had been created in a play that already holds powerful images for me. I also remember feeling pleasure, and, in retrospect, I think that this was caused by having my existing knowledge of the play challenged.

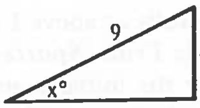
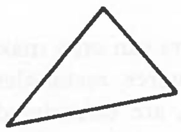
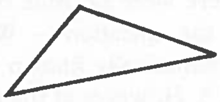
I frequently find my thoughts returning to the scene that I have just tried to describe, and as often happens I draw parallels to my world of mathematics teaching.

At the October 1994 ATM General Council weekend I put a great deal of energy into making responses to the SCAA GCSE Draft Criteria consultation exercise. One depressing aspect of this process was trying to fight against the unbending prejudices of certain politicians who are against coursework as a learning vehicle and therefore as an assessment tool. This, despite supportive comments about coursework and teacher assessment being made by both Dearing:

'... we have for too long had too limited a concept of what constitutes worthwhile achievement.' (p 43 para 5.15) and 'Teacher assessment has, therefore, an important role, ...' (p 57 para 7.6); and more recently Ofsted (Science and Mathematics in Schools – 1994):

'A sizeable lobby is growing to increase GCSE coursework in mathematics so that it contributes 40% to the results... It is rigorous and challenging to both pupils and teachers. It has moved away from its trivial 'shopping survey' activity of the early days of its introduction which, rightly, gave it a bad name. The positive impact of coursework on those who are considering mathematics at GCE A level should not be discounted' (p 24 para 3).

The draft criteria we were asked to comment on were based upon a minimum eighty percent weighting for the terminal examination. This leaves me feeling in a straight jacket, where there is pressure on me to let the examination determine the kind of teaching strategies I need to employ if my students are to achieve 'success'. My fear is that this success will be measured mostly by how they 'perform' in the examination room. Consequently,

Closed task	Modified task
$2+6-3=$	What numbers can you make from 2, 3 and 6?
$3\times5=$	Make up some questions whose answer is 15.
 Find the value of x.	Investigate what the sin button on a calculator does.
Continue this sequence 1, 2, 4 ...	Discuss how the sequence 1, 2, 4 ... might continue.
 Find the area of this triangle.	 Construct some triangles with the same area as this one.
What do we call a five-sided shape?	What shapes/configurations can you make with five lines?
Draw graphs of 1) $y=3x+5$ 2) $y=2x-4$ 3) $y=6-x$	Investigate the graphs of $y = ax + b$ for different values of a and b.
Copy and complete this addition table + | 4 | 7 2 | | 6 | |	Investigate the possible ways of completing this table: | | | 3 | 4 | 7 |

my responsibility will be subverted to teaching the students to answer narrow focused questions and spend my time in futile ways that focus on exam technique, the preparation for which does not encourage the more open learning approaches, described in Non-Statutory Guidance (page D7, see table above).

I want to be able to use my own and my colleagues' creative energies in order to find new and exciting ways of causing students to engage with mathematics so that the mathematics they do will 'be a source of delight and wonder' and will offer my students 'intellectual excitement and an appreciation of its essential creativity' (Non-Statutory Guidance, page A3, para 2.5).

The kind of narrow focused questions that examiners have been known to write when putting together a mathematics paper seem to be largely about contrivance, fragmentation, and yearly repetitions of fairly predictable questions requiring boringly closed answers. So what place is there, apart from within coursework limited to a currently derisory twenty per cent, for teachers working with students on mathematics and the unexpected?

The following sequence of events occurred in November 1993 during a PGCE session at Keele and later with another group of PGCE students at St Martin's, Lancaster. An aim for each session was to engage with number work, while at the same time interweaving investigative approaches. In the first session I had decided that I would provide students with a few situations to explore, one of which would be about factors. Just prior to the session I was mulling over how to introduce the work and noticed that I had some coloured pieces of paper. These were left-overs from some A4 paper that I had cut down to squares for some earlier work on 'limping seagulls'. Because I had previously used three different colours of paper I found myself writing the numbers from 1 to 50, one on each piece of paper, in the order of yellow, peach and blue. This meant that the yellow pieces of paper had the numbers 1, 4, 7 etc $(3n - 2)$. The peach numbers were 2, 5, 8 etc $(3n - 1)$ and the blue were the multiples of three. I was writing the last few numbers on pieces of paper as the students were entering the room.

I shared the fifty pieces of paper amongst the ten students and asked each of them to write down the factors for their five numbers. This being done, we looked at certain properties, such as: numbers with exactly two factors; numbers with an odd number of factors; who had the number with the largest number of factors. Because the numbers had been colour coded, certain interesting properties began to emerge. Next, I asked them to write on the other side of the paper all the possible consecutive number sums for each of their five numbers. The next task was to put in the middle all the numbers that didn't have a consecutive number sum. This of course, generated 1, 2, 4, 8, 16 and 32 and, as well, somebody put in 34 and 42. The nice thing was that these last two numbers were immediately rejected in a friendly and humorous way.

We noticed that all the pieces of paper were either yellow or peach – there were no blue ones. This immediately begged the question – Why? Perhaps we should have automatically known that no power of 2 has a factor of 3. However at the time this wasn't immediately obvious and we wrote the sequence 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048 down and considered the digit sum of each number. To our surprise this produced a repeating sequence of 1, 2, 4, 8, 7, 5, 1, 2, 4, . . . Later in the session we considered the digit sums of powers of five, and were further surprised to find that the reverse of the above sequence of digit sums was produced. A few months later, Andrew Mason, another student exploring a digital sums problem [1], worked out that the repeating sequence for contained the same digits as the recurring decimal for $\frac{1}{7}$, although not in exactly the same order.

More recently, an interesting phrase came out of a shape and space session with a group of PGCE (mathematics subsidiary) students at the University of Manchester. The session was based upon creating shapes by paper folding and had developed onto considerations of angles and creating tessellations. During the final fifteen-minute evaluation and feedback part of the session, we considered what the session had been about and what the implications were for classroom practice, one student (Janet) used the phrase 'expect the unexpected'. I latched on to this with a deal of excitement – expect the unexpected – what a splendid paradigm.

Another task that springs to my mind which has an unexpected set of outcomes is one I first met whilst studying with the Open University. I understand it was 'created' by Peter Gates of Nottingham University. The problem is based upon folding and cutting an isosceles right-angled triangle (irat for short) along its line of symmetry. After one fold we have two half size irats. After two folds and one cut we have...

One surprise for me is the way that the results to this problem unfold (no pun intended):

Number of folds	Number of pieces
0	1
1	2
2	3
3	4
4	6...six?

In constructing a table of results as above I am reminded of Dave Hewitt's article ***Train Spotters' Paradise*** (MT140): 'Whatever the initial mathematical situation, once the numbers are collected into a table, a separate activity begins to find patterns in the numbers. Their attention is with the numbers and is thus taken away from the original situation'.

With this problem, the numbers can only make any sense if the type of pieces (squares, rectangles, triangles) and their relative sizes, are considered. Context is therefore all important and the problem is waiting to be explored, but not as just another number-pattern spotting situation. What other surprises are held in store? What becomes predictable? An investigative approach can be applied to a problem where traditional content areas emerge. The context provides opportunities for interlinking process and content skills – types of shapes – areas of shapes – predicting the next number and types of pieces... and then there's the exam – oh well, it seems I'd better put the shapes and scissors and glue back in the cupboard, because there's no room for these in the examination hall.

If teachers are discouraged, through a narrow examination-based system, to open themselves up to these kinds of experiences and be unreceptive to the possibilities of new ideas providing new interests, then how are they going to be able to provide interesting challenges to the children they teach?

In my continuing dream, Hippolyta and Bottom become lovers and SCAA unexpectedly provide a GCSE assessment framework that endorses a flexible structure which includes a hundred per cent coursework model. Dream on...

References

1 See *Magical moments* in MT148

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ATM and the Internet

Some readers might find it useful to know that the address for the ATM Home Page is:
http://acorn.educ.nottingham.ac.uk//SchEd/pages/atm/