What is the relationship between Ma1 and the other attainment targets? **Mike Ollerton** dips into Non-statutory Guidance to find out.

CONTENT WITH PROCESS?

One of the main concerns I hear about from teachers, and particularly initial teacher trainees of mathematics, is that of being able to make Ma1 assessments of the work their students produce. This may be due to an uncertainty about how Ma1 process skills might be incorporated into teaching methodology and consequently transferred into our students' learning. Ma1 skills are often dealt with through 'bolt-on' or separate OET-type tasks and there exists a tension which is the contrast in expectations between students engaging with mathematics through process skills, yet being tested in narrow ways exclusively on content skills. I want to put forward a model which causes Ma1 skills to permeate the mathematics curriculum and at the same time provides opportunities for children to 'learn the basics'.

I believe that National Curriculum Non-Statutory Guidance provides us with not just a great deal to think about, but also some strategies by which we can develop our teaching approaches so that Ma1 becomes an integrated part of learning. In the second part of this article I offer some strategies for possible ways forward.

Extracts from and interpretations of NSG

Activities should enable pupils to communicate their mathematics

Pupils need to:
understand what needs to be done in broad terms; ...
debate possible courses of action with others; ...
present and explain results to other pupils, teachers and other adults;
make a report; ...
discuss the implications and accuracy of the conclusions reached; (B para. 5.12)

There is an image here of students being able to create and explain their mathematics. Teachers need to find strategies to make this possible. This can be achieved through a mixture of short and extended tasks. There are implications about encouraging students to share their mathematics with people other than a teacher, and parents or guardians might be encouraged to play such a role.

Activities should enable pupils to develop their personal qualities

The personal qualities which pupils need to develop include:
motivation and preparedness to tackle the unfamiliar and unknown – willingness to 'have a go';
flexibility and creative thinking in overcoming difficulties and developing new approaches;
perseverance, reliability and accuracy in working through sequences of stages in an extended task;
willingness to check, monitor and control their own work;
independence of thought and action as well as the ability to co-operate within a group;
systematic work habits. (B para. 5.13)

This demands a shift of responsibility, which is clearly not going to be achieved by teachers indicating that 'it is now up to each of you (the students) to develop further without help from me'! There must however be a whole multitude of opportunities for the students to be able to accept ever more responsibility for their learning if they are going to be able to develop such personal qualities and independence. Letting go is one of the hardest things for teachers (and parents) to do. However there must be times when learners can leave the (mathematical) nest and begin to make their own way. There are vital issues regarding teacher intervention that we need to consider and engage with, such as when it is useful to stand back and when it is useful to be didactic. Having to make instant, 'thinking on our feet' decisions is a central role for teachers. It is the making of such professional judgements that causes teaching to be draining and frustrating as well as energising and fulfilling. Of course, we are not always available to provide help and advice, and in large classes such help cannot always be immediate and so other strategies need to be considered. Once such strategy might be to ask another student in the class to take on a teaching role for a few minutes and talk to another member of the class. Group work will obviously make it easier for students to gain help and support in this way. As teachers, we all know that moment when in the course of explaining something, we have come to understand in greater depth what a certain concept really means. This also happens when one student helps another to reach an understanding about an idea.

Activities should enable pupils to develop a positive attitude to mathematics

Attitudes to foster and encourage include:
fascination with the subject;
confidence in an ability to do mathematics at an appropriate level. (B para 5.14)

Here we are encouraged to put our students in touch with the beauty and power of mathematics as a creative and imaginative subject. The final point is I believe crucial and needs to be considered both from the learner's as well as the teacher's point of view. To this end it is important that students are provided with opportunities to develop the task they are offered to whatever level they are capable. This is different to teachers predetermining that, within the same topic area or context, students of different 'abilities' need to be given different tasks to do at the beginning of a topic or piece of work.

It is not the intention of the National Curriculum to produce a narrow mechanistic approach to learning or teaching of mathematics through a rigid interpretation of the system of levels within the programmes of study. Indeed, the programmes of study which relate to using and applying mathematics, with the requirement for pupils to make choices and to work across the other elements of the programmes of study, provide a powerful disincentive to this narrow approach being adopted. (B para 7.6)

This paragraph recognises that effective learning of mathematics is not achieved by the use of a fragmented, exercise-by-exercise approach. Consequently, there is a need to construct a framework or a curriculum map, that helps teachers to decide how else the narrower skills can be taught and to offer students tasks that have the potential for developing such skills. In this way students learn to apply skills within broader problems and wider contexts than can be achieved by working through exercises from text books.

The teacher's job is to organise and provide the sort of experiences which enable pupils to construct and develop their own understanding rather than simply communicate the ways in which they themselves understand the subject (C para 2.2)

This is essentially about the important process skills through which a more controlled and deeper understanding of traditional mathematical concepts can be achieved. Showing students how to be in control of their mathematics in order to achieve ownership, rather than borrowing a set of skills which are on temporary loan from their teacher, is central to effective learning.

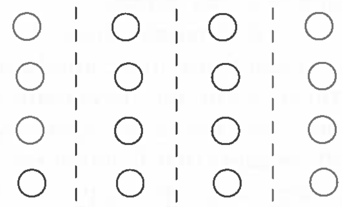
The statements of attainment within Ma1 contain the objectives for three strands of mathematical activity:

Using mathematics
Communicating mathematics
Developing ideas of argument and proof

It is through engaging in these activities that pupils will encounter the real power of mathematics. They are at the ***heart*** *of mathematics, and should underpin pupils' work across all the areas of mathematics in the programmes of study at every stage.*

This is a strong encouragement for teachers to enable students to learn the broader process skills in all the tasks they do. The occasional Ma1 tasks, bolt-on investigations or 'open extended tasks' done in controlled conditions, do not seem to fit with the intentions behind this statement. At the 1993 ATM Lancaster conference I was fortunate enough to have discussions with Peter Lacey (Professional Officer for Mathematics, SCAA) about this article and he provided the following metaphor: Ma1 is the heart of mathematics and Ma2-5 is the body. The body is easier to see but requires the heart in order to exist. With regard to assessment, the health of the heart can be determined by looking at aspects of the body. Peter also provided the following diagrams as a means for considering the interdependence of Ma2-5 with Ma1.

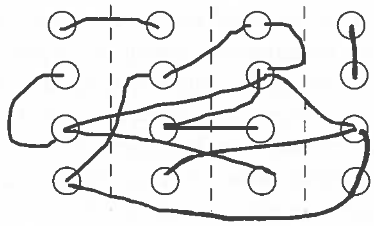
Ma2 – 5



(Destinations)

Ma2-5 skills are represented as a number of different destinations

Ma1



(Journeys)

Ma1 is about how the 'destinations' might be connected to form a map of understanding – a picture of mathematics.

Here the Ma2-5 destinations are connected by Ma1 journeys. The more journeys that are made, the more complete the map of understood mathematics becomes.

Applying mathematics to 'real life' problems

It is characteristic of 'real life' problems that they frequently do not have unique solutions; they require the selection and use of a wide range of mathematics. Applying mathematics to real problems does not come naturally or easily to many pupils, even when their grasp of the relevant knowledge and skills is sound. For this reason, pupils at all stages need to have experience of tackling 'real life' problems as an integral part of their experience of mathematics. (D para. 2.2)

A key issue therefore is transferring skills across different contexts and this is about finding ways of enabling students to gain a depth of understanding about which skills to use and when to apply them. This is different to being able to use the narrow skills that exist on page 57, exercise A, questions 1 to ... There is a further important idea within this statement which is about finding problems that cause students to learn about and acquire new skills. This is different to the strategy of teaching students a set of separate skills and then providing them with broader (convenient) problems that draw upon the taught skills. The dilemma for teachers is one of feeling that certain skills are demanded by a certain problem, and that before the students will be able to 'solve' the bigger problem they will need to be taught relevant skills. I am aware, for instance, that some teachers, before giving the *Octagon loops* task, feel they must first do some work on sequences and functions. If ever tails wagged dogs! There is also a built-in assumption that students can only perform tasks if we have first taught them. I believe that many of our students are more intelligent than we often give them credit for. Eventualities such as students applying: intuition; ideas despite my teaching; and concepts that I cannot possibly know that they already have; are all possibilities that I need to take into account.

Pupils exploring and investigating within mathematics itself

Mathematics provides a way of viewing and making sense of the real world. It is also a way of creating new imaginative worlds to explore ... this aspect of mathematics which encourages pupils to explore and explain the structure, patterns and relationships within mathematics is an important factor in enabling them to recognise and utilise the power of mathematics in solving problems and to develop their own mathematical thinking. (D para. 2.3)

Finding suitable tasks and planning how to introduce them to pupils needs to become central to teachers' preparation. Having a variety of such tasks to draw upon and recognising what works for them as teachers and is effective for their students' learning takes time and experience to develop.

The National Curriculum requires all schools to address this issue, and develop a teaching and learning approach in which the uses and applications of mathematics permeate and influence all work in mathematics. This is a major undertaking for schools, and perhaps the single and most significant challenge for the teaching of mathematics required by the National Curriculum in its aim of raising standards for all pupils. Schools must ensure that schemes of work offer sufficient opportunities for activities which specifically address the issue of using and applying mathematics. In addition, schools must carefully review their approach to teaching knowledge and skills to ensure that aspects of using, applying and investigating are integrated and embedded into the ways in which mathematics is taught and learned. (D para. 3.2)

This is probably the most powerful statement in NSG and encourages departments to look at current practice, to review methodology and to consider alternative approaches to the teaching and learning of mathematics. An example of an alternative approach is provided in the following section.

Some strategies for incorporating Ma1 skills into teaching and learning.

In order for students to be able to demonstrate that they are using and applying mathematics they need to be provided with opportunities to do so. This essentially requires a shift from teachers seeing their role exclusively as givers of knowledge, where learners are requested to carry out a collection of examples in order to show that they have received the knowledge, to other modes of teaching and learning.

Unfortunately, a teacher's role has become muddled when it comes to providing students with certain types of input. Thus, tasks such as *Octagon loops*, which have become established as the 'type' of task that teachers can use with the expectation that students responses can be matched against Ma1, are all too often seen exclusively as assessment instruments; teachers feel they cannot give too much help or tell individual students anything. This is clearly a nonsense because the teacher's role has become subverted from that of helper to one of observer and assessor. I believe that Octagon loops should be seen as just another teaching task through which assessment might be made. There can therefore be a wide range of possible teacher interventions, ranging from standing back and observing to being didactic and telling. The type of intervention, including the decision not to intervene, must be left to the teacher's professional judgement: it will be whatever best suits the learner and enhances understanding and progress.

There are many similar starter tasks which could be deemed 'open' and these can be found in resource books such as: ***Points of departure*** (Books 1, 2, 3 and 4) – published by ATM and *Starting points* (Banwell, Saunders and Tahta) published by Tarquin. These publications alone will provide well over 200 potential ideas for use in the classroom.

Determining starting points

I am more interested in constructing tasks that address a range of the Ma2-5 content skills, but are driven by Ma1 process skills. One of the more negative teaching situations that has existed within my own classroom has been where I have started working from a particular point, and, realising that many of the students had little idea of what I was talking about, I have had to take a backward step. This possibly continued until I thought I had found a point where most of the class were comfortable. By such time I had already turned off some of the less confident students and I had wasted the time of the more confident ones who were comfortable with my original starting point. Thus, I had provided a negative set of experiences for just about everyone! An alternative to this is for me at the planning stage to decide what high level area of content I might wish students to end up at and then decide what easy point I might begin from. To do this I can go further back until I feel comfortable that everyone in the group will be able to understand the beginning point. I can then offer a problem for exploration based on this beginning point. The most capable mathematicians will be expected to develop the starting problem and quickly move their thinking into more complex ideas, that I can provide further input for. This approach caters for a wide range of responses and provides learners with a positive, constructive approach to their mathematics.

For example, I want my students to explore volume. In order for them to find ways of taking control of concepts of volume, I use my knowledge of volume to find real opportunities to release my control. This must happen in a planned way. One teaching strategy is to set up a relatively simple problem and then set a problem that causes the students to explore what is happening and look for a set of results. My simple problems could be to give each pair of students 24 Multilink cubes and ask them to find all the different cuboids that can be made using all 24 cubes. I can then ask the class to calculate the surface area of each cuboid and from here I have a choice of possible ways forward.

1. Volume = 60cm^3

- explore different cuboids and surface area for V = 60;
- sketch some results on isometric paper;
- calculate the surface area of each – minimum S.A.;
- use the calculating procedure to construct a formula for surface area;
- what would the minimum surface area be if I allow non-integer lengths;
- change the shape to a cylinder, what different r and h gives V = 60;
- the above problem can be solved using a programmable calculator;
- what values of r and h give minimum surface area.

2. Volume $\leqslant$ 30 cm^3

- explore all the different cuboids that can be made for V $\leqslant$ 30; this could possibly be done in groups;
- find values which give only one cuboid (prime values);
- collect together surface areas of all the shapes that have dimensions 1 by 1 by n, 1 by 2 by n. Number patterns and the resulting formulæ can then be sought and related back to the original situation;
- collect together sets of dimensions which have the same surface area.

An important feature is that at various stages students are being encouraged to explore the ideas and to construct a meaning from the results they achieve. Again the teacher will offer varying amounts and types of input, either with individuals, with small groups or with the whole class. Thus differentiation occurs by outcome and by task.

Either approach can lead to students writing up their findings with explanations, diagrams, tables of results, graphs, formulæ and possibly a final section where students are asked to reflect upon their learning. This could be a list of ideas that they: know at the end of the project; had been reminded of; didn't know or had forgotten prior to starting the work. This can then link in with a student's record of achievement.

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MICROMATH
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In February 1994 the General Council of the ATM will be choosing a team of at least 2 people to edit **Micromath**. The appointment will be for 3 years in the first instance and is only open to personal members of the Association.

It is envisaged that the new editorial team would assume responsibility for the publication from the Spring 1995 issue. However, it is expected that the appointees will start their involvement soon after the appointment.

An honorarium is paid for the editorship of each issue. In 1994 this will be £1500 per issue. The editors negotiate a total annual budget with the General Council and work in conjunction with a designer prior to publication by the ATM.

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The closing date for application is Friday 14th January 1994