SEEDING

OR WAYS OF BUILDING ASSESSABLE TASKS INTO NORMAL COURSE ACTIVITIES

Mike Ollerton

Orleton Park School is one of the six schools which currently works with a GCSE Mathematics syllabus developed by the Association of Teachers of Mathematics and the Southern Examining Group. The syllabus was originally designed to use a 100% coursework assessment model and in 1988 and 1989 students were certificated accordingly. In order for the syllabus to conform to the present National Criteria some rationalisation had to take place and the present assessment model is 50% coursework, where content is taught directly through process, a 25% process examination and a 25% conventional content examination. The process component requires students to answer three questions in a 2 hour exam and their responses are graded using two of the syllabus assessment criteria; implementation and interpretation.

Seeking opportunities through coursework for the encouragement of differentiation by outcome underpins the way a piece of work is started and developed. The assessment of the work and the comments that teachers make, both verbal and written, are integral features of the way students are helped to understand how well they have worked on a piece of mathematics, and what they can do in order to improve in the future.

In order to enable differentiation by outcome, the teacher must consider how a piece of mathematics can be made accessible to all the students whilst at the same time knowing that the most able students will be able to develop the idea into more complex areas of the content and deal with the concepts at more sophisticated levels. To achieve this it is important to 'trace' back the ideas contained within a high level of content and look for the 'seed' that can be used to help students to produce a piece of mathematics. In order to illustrate this I have chosen *transformation matrices* as the high level area of content for students to aim for. The 'seed' is to use reflections and rotations of shapes drawn on a co-ordinate grid. How simplistic the teacher decides to make the initial starting task will depend upon the knowledge that a teacher has about their class. For the following piece of work it may be desirable to use practical equipment such as pegs and pegboards in order to perform the initial transformations. Reminding all students of the convention of plotting a co-ordinate pair by moving horizontally first and vertically second, whilst unnecessary for them all, only takes a few moments to mention. (The students will have worked on modules based upon co-ordinates in their first and second year, and a module on vectors in their third year).

The story line is therefore:

a cartesian co-ordinates [AT7/4a][AT7/5a] [AT11/4a][AT11/5c]

b vectors [AT11/8b]

c reflection symmetry [AT11/3a][AT11/6b]

d rotational symmetry [AT11/4b]

e enlargements [AT11/6c][AT11/7b]

f combinations of transformations [AT11/10a]

g equations of axes and other 'simple' straight lines [AT6/5a]

h matrices [AT11/10b]

Three important features are that the students are working in an investigative style, and the teacher makes decisions about when to intervene and when to allow students to take more control of the problem. The teacher has to decide upon the kind of discussions and inputs that would be useful for different students, either to move a student on, or to suggest a slightly easier task within the framework. What is important is that students progress *along the story line* according to their ability and interest. This is different to a student proceeding upon a line of enquiry explicitly set out by a published text where students are responding to tightly defined exercises. The third feature is that students are working at different levels across different attainment targets as set out in the National Curriculum document.

The following notes were written for teachers in the department. Each of the activities are not 'delivered' to the whole class, except for the initial introduction, where the problem is set up and guidelines are explained. The first set of notes are the ones presently used and written for a mixed ability class. The activities will be discussed and students could take rough notes.

Transformations

5th Year 1990/91

Activity 1

On a co-ordinate grid reflect a shape of your choosing in the:

a) $y = 0$ line
b) $x = 0$ line
c) $y = x$ line
d) $y = -x$ line

Obviously the terminology will need explaining; you may wish to talk about reflecting in the x and y axis and leave any mention of the equations of these lines until later.

For each of a) b) c) and d) see how the co-ordinates change (written in vector notation)

eg under b) $\begin{pmatrix} X_1, & X_2, & X_3, & X_4 \\ Y_1, & Y_2, & Y_3, & Y_4 \end{pmatrix}$

becomes $\begin{pmatrix} -X_1, & -X_2, & -X_3, & -X_4 \\ Y_1, & Y_2, & Y_3, & Y_4 \end{pmatrix}$

A simple conclusion can be drawn.

Now consider a shape that crosses the y axis ...

Activity 2

Consider rotations 90°, 180°, 270°, 360° anti-clockwise ...

Again see how the co-ordinates change.

Activity 3

Consider combinations of rotations and reflections.

Activity 4

Enlargements from (0,0) and other centres.

Negative enlargements, fractional enlargements.

Activity 5

By this point (somewhere into the third lesson) some of the students (the 4 or 5 most able in the group), will be ready to move onto a more complex level of thinking. Those who are 'ready' to take on board the idea of transforming using matrices will need the appropriate instruction about how to carry out such a calculation.

Consider the matrix transformation $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(Appropriate instruction at this point will mean the teacher giving the students a clear 'didactic' explanation about how to multiply by a 2 by 2 matrix. The important skill for the teacher is in recognising which students need to be moved on.)

Which transformation from before gives the same result?

What matrix transformations are needed to produce the other transformations produced earlier?

Activity 6

Consider other matrix transformations using (-1, 0, 1) for a, b, c, d for 2 by 2 matrices of the form $\begin{pmatrix} a & b \\ d & c \end{pmatrix}$.

This activity is described more fully in the notes below.

For the next three weeks the students will be engaging with various tasks. The students usually work in rough in an exercise book and as they proceed with a task they are encouraged to discuss with each other the work they are doing. Between a half and two thirds of the group will have voluntary discussions with the teacher. Those more reluctant to talk about their mathematics are asked to bring their work out and tell the teacher what they have done, where they have got to, what they understand, what they are struggling with and where they intend to go next with the problem.

A useful strategy, once a dozen or so students have begun working with matrices, is to ask each person to produce an A4 size diagram on graph paper showing the original shape, a transformation and the accompanying matrix. These can then be mounted on the wall as an instant display, and all students can have access to a lot of readily available information for further analysis.

The rate of progress that each student makes affects the decisions that the teacher makes about subsequent action. During this period the teacher will not do any formal marking of the work (ie won't collect the students' books in), however the teacher will have formed a clear picture of where each student has progressed to with the work through various conversations with each student over this three week period. At the end of the time made available for the work, the students hand in their completed write-ups and it is at this point that the teacher has a heavy marking load. However the type of assessments that are made and the comments that are written back to the students are more meaningful than the type of marking that was formally done, (ie the books collected in once a week and returned to students with a series of ticks, possibly with a short comment).

In responding to a piece of coursework the teacher will write a grade for each of the seven categories that apply to the work that the student has produced. The seven assessment criteria written in to the syllabus are: communication, implementation, mathematical knowledge, interpretation, evaluation, mathe-matical attitude and autonomy. In addition to this, comments will be written on the students script as well as a summative comment on the cover/grade sheet. Students are therefore positively en-couraged to work on the processes by the nature of the assessment.

The next set of teacher notes were those written for top set classes only. The implication is that only these students were entitled to work in this area of mathematics, so most students (ie all those not in the top set) were being limited from the outset.

Matrix transformations

5th Year (set 1) 1988/89

This is a content-based investigation using 2 by 2 matrices to transform simple shapes on a coordinate grid. The work can be done in small groups.

I started off by going through the mechanics of:

1) multiplying matrices
2) writing the co-ordinates of a shape in vector array
3) transforming a shape by a matrix
4) describing the resulting transformation.

Having spent the best part of a double lesson going through these procedures, I set the problem of finding all the different transformations using matrices of the form

$$\begin{pmatrix} a & b \\ d & c \end{pmatrix}$$

where a, b, c and d can take the values -1, 0 or 1

The work can lead to some very interesting ways of interpreting and classifying information.

I suggested that starting with a rectangle would help with determining what the resulting transformations were.

After a further two or three lessons I suggested to some of the most able that they have a look at which matrices 'undid' themselves, and this led to some students working on inverses.

If the group theory work has already been tackled, references to which matrices form a group can be made. If not, then this work can be recalled when the work on groups is tackled.

One effect that working with this syllabus has had upon the department has been to develop mixed ability teaching systems with upper school GCSE classes. I emphasise teaching systems rather than teaching groups because I believe that to group children in mixed ability classes and then offer an individualised workcard or work book scheme does little to enhance learning, particularly teachers' or students' personal responsibilities towards learning. The move to mixed ability in mathematics classes at the school has been brought about largely because staff recognised, through the vehicle of coursework, that students were encouraged to respond to stimuli according to their ability rather than to a pre-conceived notion which was seemingly determined according to the maths set that students had been placed in at the end of the students' third year or indeed to gender. ■

Orleton Park School, Telford

(Mike Ollerton is the chair of the ATM/SEG GCSE Steering Group. The article was originally written as a paper for a one-day conference on coursework organised by the SEAC – Eds)

THE WEATHER

Marjorie Gorman

The weather is an ever-ready resource for data collection from the earliest year. The records opposite were made by a reception class at Girnhill Infant School, Featherstone.

At the start of the school year observations were made at some time during the morning and during the afternoon. By July, recording was pictorial. Over the course of the year it became apparent to them that increasing accuracy in the taking and recording of observations was needed. Recording was more detailed and written. Their teacher, Miss Norah Carlisle recognised that carefully kept records like these form a useful resource for the school.

Observations of weather conditions and discussion about them were part of the daily routine of a group of middle/top infants at Bell Lane First School, Ackworth.

The idea of arranging the observations on a 'washing line' came from their teacher, Mrs Jennie Cooke, who had seen a similar arrangement with numbers when attending a local in-service course. (See double-page spread). Responsibility for collecting week-end observations was shared amongst the class and a differently shaped card was used to differentiate between 'school' days and 'non-school' days.

The children were able to arrange the cards on the 'washing line' to answer such questions as:

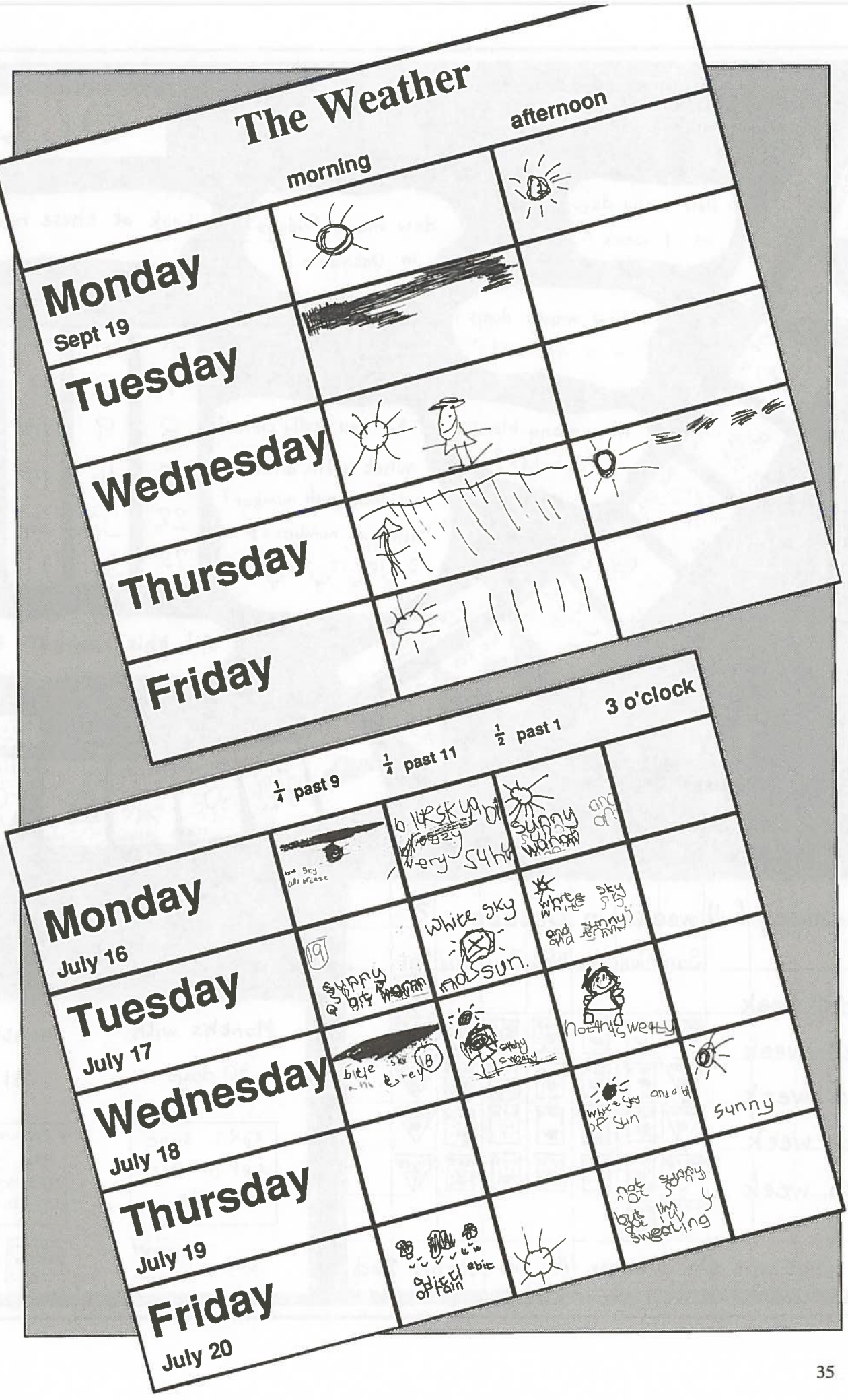
- How many rainy days?
- How many rainy Sundays?
- How many rainy Mondays?
- Were there more rainy days than sunny days?

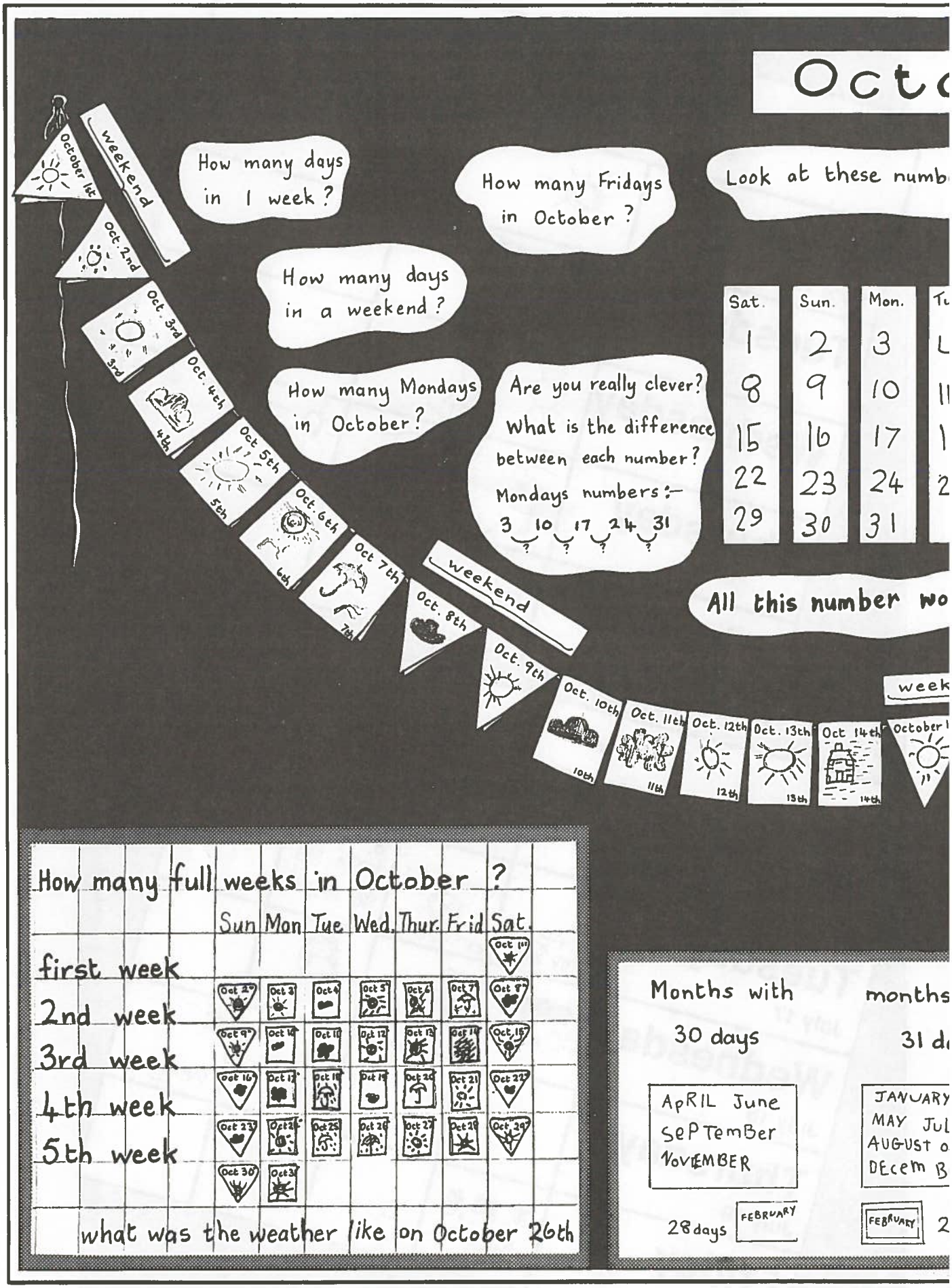
The children made up their own questions and the linear arrangement was in contrast with the more usual calendar arrangement. The work was extended into work on calendar numbers and counting in sevens.

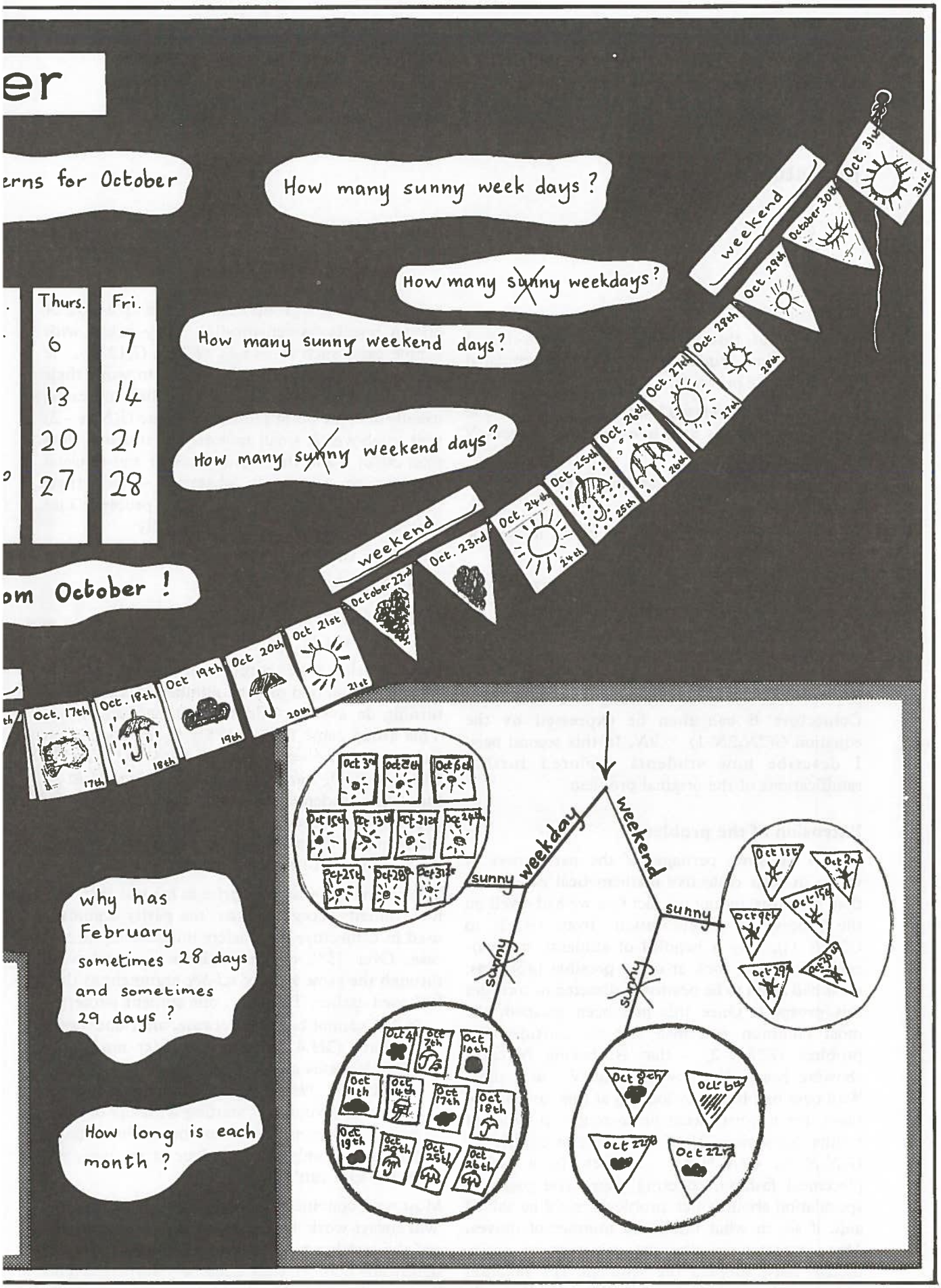
The double-page spread over the page is an artist's impression of a display of their work put up in Wakefield Teachers' Centre last October for a course, called Infant Maths week, for teachers of Key Stage 1 children.

Beginnings of handling data in the National Curriculum! ■

Advisory Teacher, Wakefield


The Weather
morning
afternoon
Monday
Sept 19
Tuesday
Wednesday
Thursday
Friday
¼ past 9
¼ past 11
½ past 1
3 o'clock
Monday
July 16
Tuesday
July 17
Wednesday
July 18
Thursday
July 19
Friday
July 20
White sky
and sun.
sunny
Sunny


Octo
How many days in 1 week?
How many days in a weekend?
How many Mondays in October?
How many Fridays in October?
Look at these numb
Are you really clever?
What is the difference between each number?
Mondays numbers:-
3 10 17 24 31
Sat. Sun. Mon.
1 2 3
8 9 10
15 16 17
22 23 24
29 30 31
All this number wo
weekend
weekend
October 1st
Oct. 2nd
Oct. 3rd
Oct. 4th
Oct. 5th
Oct. 6th
Oct 7th
Oct. 8th
Oct. 9th
Oct. 10th
Oct. 11th
Oct. 12th
Oct. 13th
Oct 14th
How many full weeks in October ?
Sun Mon Tue Wed Thur Frid Sat
first week
2nd week
3rd week
4th week
5th week
what was the weather like on October 26th
Months with 30 days
APRIL June SEPTEMBER NOVEMBER
28 days FEBRUARY
months
31 d
JANUARY MAY JUL AUGUST O DECEM B
FEBRUARY 2



Zena Ginifer from a photograph by Mrs Kay Sewell, who also put up the display.