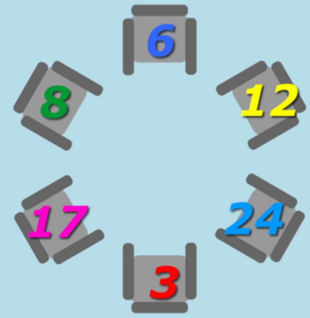
Being A Number



By Mike Ollerton 06 May 2012

INTRODUCTION

I have used each of the following ideas many times with different age groups of people in a variety of contexts.

Sitting in a ring of chairs and giving everyone a number …

MATHEMATICAL CONTENT

- Properties of numbers
- Sequences
- Algebra

APPLICABILITY

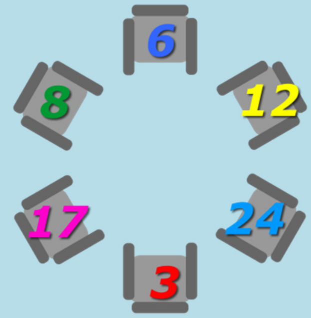
KS1 – KS5

1 **What's my number?**

If the numbers are given out in random order and students are asked to keep their number a 'secret', one starting problem could be to ask each person to construct a calculation such that their number is the answer to their calculation. They could be asked to create 2- or 3-stage calculations. There are opportunities here to develop ever-more sophisticated concepts such as squaring, cubing, factorals. For example if my number is 17 then one calculation could be 4! subtract 2^3 add 1. Of course students could be invited to work with someone sitting next to them; this is in order to support those students who may not respond to the pressure of having to create a calculation.

2 **Swapping places**

All those who are a multiple of 3 swap places
All those with a square number swap places
All those with a Fibonacci number swap places
All those who are 2 less than a multiple of 5 swap places
After a few goes, I remove my chair… then things get a little more hectic !



3 **Every other person sit down**

Starting with everybody standing up and in order from 1 to n, this problem begins with number 1 sitting down then every other person subsequently sits down. Thus the first time around the circle numbers 1, 3, 5, 7 etc will sit down thus leaving the even numbers standing.

On the second time around, depending upon how many people there were in the original circle, the next sequence will either be: 2, 6, 10, 14… etc or 4, 8, 12, 16… etc.

The problem is to see explore who is the last person to sit down. Trying to construct a way of working out what number will remain at the end according to how many people there were to beginning can be a pattern spotting exercise for younger learners. However, constructing a formula to describe the mapping would challenge Y12 and older students.

Developing the problem to every third person sits down is something I once played around with on a long train journey. I have not reached a solution… yet!

4 **Generating sequences and exploring intersecting sequences**

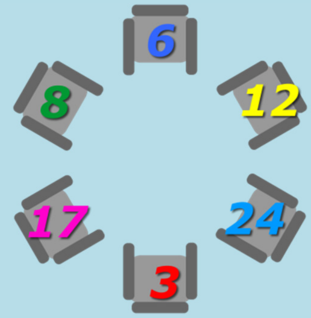
The idea is to find the person who is, say, 2 more than their number. Everyone apart from the person with the highest two numbers will be able to find another person whilst everyone will be ‘found’.

In effect the group will be partitioned into the even (2n) numbers and the odd (2n – 1) numbers. Students write one of these expressions on the reverse of their card.

This procedure is repeated but this time the idea is to find the person who is 3 more than your number. The sequences 3m, 3m – 1 and 3m – 2 will be achieved this time. Again students record which group they are in on the back of their card.

I then ask students to group together according to whether they have the same pair of expressions on their cards. This creates six groups,

For Y12+ learners they can explore how to find single expressions for intersecting pairs of sequences, e.g. which people were both in the sequence 2n – 1 and a 3m – 2?
This can easily be extended to consider other pairs of intersecting sequences, e.g. the intersection of 3n – 2 with 4m – 1.



5 **Cuisenaire Arithmetic**

This idea is taken from "Some lessons in mathematics: A handbook on the teaching of 'Modern Mathematics'" by members of the Association of Teachers of Mathematics (www.atm.org.uk) edited by T, J, Fletcher (1964), pp 44-46

Everybody sits in a ring of chairs (all desks pushed to the back or around the sides of the room) and is given a number from 1 upwards and a Cuisenaire rod according to the following modular sequence:
number 1 is given a white rod,
number 2 is given a red rod,
number 3 is given a pale green rod,
number 4 is given a pink rod,
number 5 is given a yellow rod.
This sequence of colours then repeats, do 6 is white, 7 is red etc.

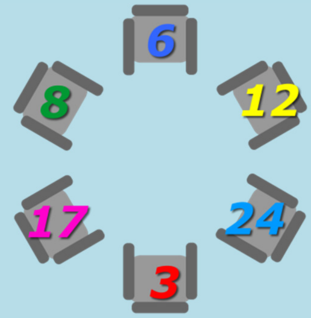
Once everybody has a number I ask all those holding a white to stand up and call out their numbers (1, 6, 11, 16. 21 etc)

Some question can be:
What does anyone notice about these numbers?
What colour would 36 be? What colour 51 be? What would be the highest number, with a white rod if there were a hundred, a thousand or a million people in the circle?

In turn all the Reds, Greens, Pinks and Yellows stand up and call out their numbers.

The teacher might choose to record the results on the board:
White Red Green Pink Yellow
1 2 3 4 5
6 7 8 9 10
11 12 13 etc etc

Continues…/



5 **Cuisenaire Arithmetic**

Continued…/

There clear opportunities for pattern spotting here etc.

Further situations might be to ask a red and a pink to stand up and see what colour their sum is. Ask a different red and pink to stand up – what colour is their sum?
I do this a few times until the idea is becoming accepted that the equation r + p = w can be written. Why does this happen?

What happens if we add two different colours? What will the corresponding equations be? What happens if I add a pink to a pink? What is the equation now?

? What happens if we triple a green?
? What happens if we multiply a pink by a green
? What happens with other multiplications?
? What about red squared?
? How about r x (p + y)?

At some point in proceedings the group can reform the classroom, or, if the desks have been set out in such a way that the ring of chairs can be turned around to work at the desks around the edge of the room, then we have another interesting classroom arrangement of furniture for the 'remainder' of the lesson!

The next task is to produce addition and multiplication tables (modulo 5).

Students may prefer to colour the spaces in the tables leading to an effective display.

What happens when we consider subtraction? (Obviously the issue of non-commutativity of subtraction needs to be considered here)

Students can be encouraged to think about what happens if we only have 4 colours instead of 5. What happens to the tables now?

What happens with 6, 7, 8, 9, 10 etc colours. All these addition and multiplication tables can be displayed and students can look for general rules about the structure within the tables.